出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长，学习他的科学思想和卓越风范，展示其丰功伟绩和人格魅力，西安交通大学举办了“纪念钱学森诞辰100周年”系列活动：作为制片方之一，参与西部电影集团摄制传记故事片《钱学森》；与中央电视台合作，出品纪录片《实验班的故事——沿着钱学森走过的路》；扩建钱学森生平业绩展馆，向校内外开放；举办钱学森科学与教育思想研讨会；出版发行《钱学森力学手稿》、《钱学森年谱（初编）》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿，这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风，是钱老勇攀科学高峰和严谨治学的集中体现。这里，我们将部分原稿整理汇集成册，出版《钱学森力学手稿》，作为钱老百年诞辰的献礼。

《钱学森力学手稿》共10卷，包含两部分内容。第一部分是草稿，包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时，为了求得圆柱壳体的临界压力，在有关的五百多页草稿中，对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算，最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表，每个数字有效位数都长达八位，在手摇机械式计算机作为主要计算工具的年代，这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿，以航空航天工程为核心，涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下，由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助，在此深表感谢。

Preliminary Calculation of Circular Cylinder (V)

$$\frac{w}{R} = \left(f_0 + \frac{1}{4}f_1\right) + \frac{1}{2}f_1\left[\cos\frac{mx}{R}\cos\frac{my}{R} + \frac{1}{4}\cos\frac{2mx}{R} + \frac{1}{4}\cos\frac{2my}{R}\right]$$

$$+ \frac{1}{4}f_2\left[\cos\frac{2mx}{R} + \cos\frac{2my}{R}\right]$$

$$\frac{w}{R} = \left(f_0 + \frac{1}{4}f_1\right) + \frac{1}{2}f_1\cos\frac{mx}{R}\cos\frac{my}{R} + \frac{1}{4}\left(\frac{1}{2}f_1 + f_2\right)\cos\frac{2mx}{R} + \frac{1}{4}\left(\frac{1}{2}f_1 + f_2\right)\cos\frac{2my}{R}$$

$$\frac{\partial w}{\partial y} = -m\left[\frac{1}{2}f_1\cos\frac{mx}{R}\sin\frac{my}{2} + \frac{1}{2}\left(\frac{1}{2}f_1 + f_2\right)\sin\frac{2my}{R}\right]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2\left[\frac{1}{2}f_1\cos\frac{mx}{R}\cos\frac{my}{R} + \left(\frac{1}{2}f_1 + f_2\right)\cos\frac{2mx}{R}\right]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial y^2} = -\left(\frac{m}{R}\right)^2\left[\frac{1}{2}f_1\cos\frac{mx}{R}\cos\frac{my}{R} + \left(\frac{1}{2}f_1 + f_2\right)\cos\frac{2my}{R}\right]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x\partial y} = +\left(\frac{m}{R}\right)^2\left[\frac{1}{2}f_1\sin\frac{mx}{R}\sin\frac{my}{R}\right]$$

$$\Delta\Delta F = E\left(\frac{m}{R}\right)^2\left[m^2\left\{-\frac{1}{8}f_1^2\left(\cos\frac{2mx}{R} + \cos\frac{2my}{R}\right) - \frac{1}{4}f_1\left(\frac{1}{2}f_1 + f_2\right)\left(\cos\frac{mx}{R} + \cos\frac{3mx}{R}\right)\cos\frac{my}{R}\right.\right.$$

$$\left. - \frac{1}{4}f_1\left(\frac{1}{2}f_1 + f_2\right)\cos\frac{mx}{R}\left(\cos\frac{my}{R} + \cos\frac{3my}{R}\right) - \left(\frac{1}{2}f_1 + f_2\right)^2\cos\frac{2mx}{R}\cos\frac{2my}{R}\right\}$$

$$\left. + \frac{1}{2}f_1\cos\frac{mx}{R}\cos\frac{my}{R} + \left(\frac{1}{2}f_1 + f_2\right)\cos\frac{2mx}{R}\right]$$

$$= E\left(\frac{m}{R}\right)^2\left[-\left\{\frac{1}{8}f_1^2 m^2 - \left(g - \frac{1}{2}f_1\right)\right\}\cos\frac{2mx}{R} - \frac{1}{8}f_1^2 m^2\cos\frac{2my}{R}\right.$$

$$- \left\{\frac{1}{2}f_1\left(g - \frac{1}{2}f_1\right)m^2 - \frac{1}{2}f_1\right\}\cos\frac{mx}{R}\cos\frac{my}{R} - \frac{1}{4}f_1\left(g - \frac{1}{2}f_1\right)m^2\cos\frac{3mx}{R}\cos\frac{my}{R}$$

$$\left. - \frac{1}{4}f_1\left(g - \frac{1}{2}f_1\right)m^2\cos\frac{mx}{R}\cos\frac{3my}{R} - \left(g - \frac{1}{2}f_1\right)^2 m^2\cos\frac{2mx}{R}\cos\frac{2my}{R}\right]$$

$$F = E\left(\frac{R}{m}\right)^2\left[-\frac{1}{16}\left\{\frac{1}{8}f_1^2m^2-\left(g-\frac{1}{2}f_1\right)\right\}\cos\frac{2mx}{R}-\frac{1}{128}f_1^2m^2\cos\frac{2my}{R}-\frac{1}{4}\left\{\frac{1}{2}f_1\left(g-\frac{1}{2}f_1\right)m^2-\frac{1}{2}f_1\right\}\cos\frac{mx}{R}\cos\frac{my}{R}\right.$$

$$\left.-\frac{1}{400}f_1\left(g-\frac{1}{2}f_1\right)m^2\cos\frac{3mx}{R}\cos\frac{my}{R}-\frac{1}{400}f_1\left(g-\frac{1}{2}f_1\right)m^2\cos\frac{mx}{R}\cos\frac{3my}{R}-\frac{1}{64}\left(g-\frac{1}{2}f_1\right)^2m^2\cos\frac{2mx}{R}\cos\frac{2my}{R}\right]$$

$$\sigma_x+\sigma_y = E\left[\frac{1}{4}\left\{\frac{1}{8}f_1^2m^2-\left(g-\frac{1}{2}f_1\right)\right\}\cos\frac{2mx}{R}+\frac{1}{32}f_1^2m^2\cos\frac{2my}{R}+\frac{1}{2}\left\{\frac{1}{2}f_1\left(g-\frac{1}{2}f_1\right)m^2-\frac{1}{2}f_1\right\}\cos\frac{mx}{R}\cos\frac{my}{R}\right.$$

$$\left.+\frac{1}{40}f_1\left(g-\frac{1}{2}f_1\right)m^2\cos\frac{3mx}{R}\cos\frac{my}{R}+\frac{1}{40}f_1\left(g-\frac{1}{2}f_1\right)m^2\cos\frac{mx}{R}\cos\frac{3my}{R}+\frac{1}{8}\left(g-\frac{1}{2}f_1\right)^2m^2\cos\frac{2mx}{R}\cos\frac{2my}{R}\right]$$

$$\lambda+4\frac{\sigma}{E}-\frac{1}{2}m^2\left[\frac{1}{16}f_1^2+\frac{1}{8}\left(g-\frac{1}{2}f_1\right)^2\right]+\left(f_0+\frac{1}{4}f_1\right)=0$$

$$\boxed{K=-4\left(\frac{\sigma}{E}\right)^2-m^2\frac{\sigma}{E}\left[\frac{1}{4}f_1^2+\frac{1}{2}\left(g-\frac{1}{2}f_1\right)^2\right]}$$

$$P_1=\frac{1}{8}\left\{\frac{1}{8}f_1^2m^2-\left(g-\frac{1}{2}f_1\right)\right\}^2+\frac{1}{512}f_1^4m^4+\frac{1}{16}\left\{f_1\left(g-\frac{1}{2}f_1\right)m^2-f_1\right\}^2+\frac{1}{800}f_1^2m^4\left(g-\frac{1}{2}f_1\right)^2$$

$$+\frac{1}{64}m^4\left(g-\frac{1}{2}f_1\right)^4$$

$$p_1 = \frac{1}{8}\Big\{ \frac{1}{64} f_1^4 m^4 - \frac{1}{4} f_1 m^2 (g - \tfrac{1}{2} f_1) + (g - \tfrac{1}{2} f_1)^2 + \frac{1}{64} f_1^4 m^4 + \frac{1}{2} f_1^2 m^4 (g - \tfrac{1}{2} f_1)^2 - f_1^2 m^2 (g - \tfrac{1}{2} f_1)$$

$$+ \frac{1}{2} f_1^2 + \frac{1}{100} f_1^2 m^4 (g^2 - g f_1 + \tfrac{1}{4} f_1^2) + \frac{1}{8} m^4 (g^4 - 2 g^3 f_1 + \tfrac{3}{2} g^2 f_1^2 - \tfrac{1}{2} g f_1^3 + \tfrac{1}{16} f_1^4) \Big\}$$

$$= \frac{1}{8}\Big\{ \frac{1}{64} f_1^4 m^4 - \frac{1}{4} f_1^2 g m^2 + \frac{1}{8} f_1^3 m^2 + g^2 - g f_1 + \frac{1}{4} f_1^2 + \frac{1}{64} f_1^4 m^4 + \frac{1}{2} f_1^2 g^2 m^4 - \frac{1}{2} f_1^3 g m^4$$

$$+ \frac{1}{8} f_1^4 m^4 - f_1^2 g m^2 + \frac{1}{2} f_1^3 m^2 + \frac{1}{2} f_1^2 + \frac{1}{100} f_1^2 g^2 m^4 - \frac{1}{100} f_1^3 g m^4 + \frac{1}{400} f_1^4 m^4$$

$$+ \frac{1}{8} g^4 m^4 - \frac{1}{4} f_1 g^3 m^4 + \frac{3}{16} f_1^2 g^2 m^4 - \frac{1}{16} f_1^3 g m^4 + \frac{1}{128} f_1^4 m^4 \Big\}$$

$$= \frac{1}{8}\Big[m^4 \Big\{ \big(\tfrac{1}{64} + \tfrac{1}{64} + \tfrac{1}{8} + \tfrac{1}{400} + \tfrac{1}{128}\big) f_1^4 + \big(-\tfrac{1}{2} - \tfrac{1}{100} - \tfrac{1}{16}\big) f_1^3 g + \big(\tfrac{1}{2} + \tfrac{1}{100} + \tfrac{3}{16}\big) f_1^2 g^2 - \tfrac{1}{4} f_1 g^3 + \tfrac{1}{8} g^4 \Big\}$$

$$- m^2 \Big\{ \big(-\tfrac{1}{8} - \tfrac{1}{2}\big) f_1^3 + \big(\tfrac{1}{4} + 1\big) f_1^2 g \Big\} + \Big\{ \tfrac{3}{4} f_1^2 - g f_1 + g^2 \Big\} \Big]$$

$$\boxed{\begin{aligned} p_1 = \frac{1}{8}\Big[& m^4 \Big\{ \frac{533}{3200} f_1^4 - \frac{229}{400} f_1^3 g + \frac{279}{400} f_1^2 g^2 - \frac{1}{4} f_1 g^3 + \frac{1}{8} g^4 \Big\} \\ & + m^2 \Big\{ \frac{5}{8} f_1^3 - \frac{5}{4} f_1^2 g \Big\} + \Big\{ \frac{3}{4} f_1^2 - g f_1 + g^2 \Big\} \Big] \end{aligned}}$$

$$\delta_2 = \frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \left[f_1^2 + 4\left(g - \frac{1}{2}f_1\right)^2 \right] = \frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \left[2f_1^2 + 4g^2 - 4f_1 g \right]$$

$$\delta_2 = \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \left[f_1^2 - 2f_1 g + 2g^2 \right]$$

$$\eta = \left(\frac{g}{t}\right) = \frac{g}{\left(\frac{t}{R}\right)}$$

$$\rho = \frac{f_1}{g}$$

$$K = -4\left(\frac{\sigma}{E}\right)^2 - m^2 \frac{\sigma}{E}\left[\frac{3}{8}f_1^2 - \frac{1}{2}gf_1 + \frac{1}{2}g^2 \right]$$

$$\frac{\sigma R}{Et}\gamma\left(\frac{3}{4}\rho - \frac{1}{2}\right) = \frac{1}{8}\Big[(\gamma\eta)^2\left(\frac{533}{800}\rho^3 - \frac{687}{400}\rho^2 + \frac{279}{200}\rho - \frac{1}{4}\right) + (\gamma\eta)\left(\frac{15}{8}\rho^2 - \frac{5}{2}\rho\right)$$
$$+ \left(\frac{3}{2}\rho - 1\right)\Big] + \frac{1}{3(1-\nu^2)}\gamma^2(\rho - 1)$$

$$\frac{\sigma R}{Et}\gamma\left(\frac{1}{2}\rho - 1\right) = \frac{1}{8}\Big[(\gamma\eta)^2\left(\frac{229}{400}\rho^3 - \frac{279}{200}\rho^2 + \frac{3}{4}\rho - \frac{1}{2}\right) + (\gamma\eta)\left(\frac{5}{4}\rho^2\right) + (\rho - 2)\Big]$$
$$+ \frac{1}{3(1-\nu^2)}\gamma^2(\rho - 2)$$

$\frac{10}{16}$ $\frac{10}{8}$

$\frac{9}{4}$

$$\frac{\sigma R}{Et}\gamma(3\rho-2) = (\gamma\eta)^2\left(\frac{533}{1600}\rho^3 - \frac{687}{800}\rho^2 + \frac{279}{400}\rho - \frac{1}{8}\right) + (\gamma\eta)\left(\frac{15}{16}\rho^2 - \frac{5}{4}\rho\right) + \left(\frac{3}{4}\rho - \frac{1}{2}\right) + \frac{2}{3(1-\nu^2)}\gamma^2(2\rho-2)$$

$$\frac{\sigma R}{Et}\gamma(\rho-2) = (\gamma\eta)^2\left(\frac{229}{1600}\rho^3 - \frac{279}{800}\rho^2 + \frac{75}{400}\rho - \frac{1}{8}\right) + (\gamma\eta)\left(\frac{5}{16}\rho^2 + 0\right) + \left(\frac{1}{4}\rho - \frac{1}{2}\right) + \frac{2}{3(1-\nu^2)}\gamma^2(\rho-2)$$

$$0 = (\gamma\eta)^2\left(\frac{154}{1600}\rho^4 - \frac{150}{800}\rho^3 - \frac{54}{400}\rho^2 - \frac{25}{100}\rho\right) + (\gamma\eta)\left(\frac{5}{4}\rho^2 + 0\right) + (-\rho + 0) + \frac{2}{3(1-\nu^2)}\gamma^2(\rho^2 - 4\rho)$$

$$+ (\gamma\eta)^2\left(+ \frac{304}{800}\rho^3 - \frac{408}{400}\rho^2 + \frac{102}{100}\rho\right) + (\gamma\eta)\left(\frac{5}{4}\rho^2 - \frac{5}{2}\rho\right) + (\rho) + \frac{2}{3(1-\nu^2)}\gamma^2(+4\rho)$$

$$0 = (\gamma\eta)^2\left(\frac{77}{800}\rho^3 + \frac{77}{400}\rho^2 - \frac{231}{200}\rho + \frac{77}{100}\right) + (\gamma\eta)\left(\frac{5}{2}\rho - \frac{5}{2}\right) + \frac{2}{3(1-\nu^2)}\gamma^2(\rho - 2)$$

$$\left\{\frac{77}{800}(\gamma\eta)^2\right\}\rho^3 + \left\{\frac{77}{400}(\gamma\eta)^2\right\}\rho^2 + \left\{-\frac{231}{200}(\gamma\eta)^2 + \frac{5}{2}(\gamma\eta) + \frac{2}{3(1-\nu^2)}\gamma^2\right\}\rho + \left\{\frac{77}{100}(\gamma\eta)^2 - \frac{5}{2}(\gamma\eta) - \frac{4}{3(1-\nu^2)}\gamma^2\right\} = 0$$

$\gamma = 0.10, \quad \eta = 10, \ \xi = 10.5051 \ \gamma\eta = 1$

$\lambda = -0.0480814$

$0.09625 \rho^3 + 0.19250 \rho^2 + 1.352326 \rho - 1.744652 = 0$

$F(\rho) - \rho^3 + 2.00000 \rho^2 + 14.05014 \rho - 18.12625 = 0$

$F'(\rho) = 3\rho^2 + 4.00000 \rho + 14.05014$

$F(1.05) = -0.010978 \qquad F'(1.05) = 21.558$

$\underline{0.00051}$

$F(1.05051) = 0$

$\rho^2 + 3.05051 \rho + 17.2547 = 0$ has no Real Root !!!

$\boxed{\rho = 1.05051}, \quad \rho^2 = 1.10357, \quad \rho^3 = 1.15931$

$$\frac{\sigma R}{Et} = \frac{2}{3(1-\nu^2)} \gamma + \frac{1}{\gamma(\rho-2)} \left\{ (\gamma\eta)^2 (0.143125 \rho^3 - 0.34875 \rho^2 + 0.1875 \rho - 0.1250) + (\gamma\eta) 0.3125 \rho^2 \right\} + \frac{1}{4\gamma}$$

for this particular case

$$\frac{\sigma R}{Et} = 0.07326 + 10 \left\{ \frac{0.143125 \rho^3 - 0.03625 \rho^2 + 0.1875 \rho - 0.1250}{-0.94949} + 0.25 \right\}$$

$$= 0.07326 + 0.41580 = \underline{\underline{0.4891}}$$

$$(\delta\xi) = 1.05051$$

$$(\delta\xi)^2 = 1.103571$$

$$\dot{\varepsilon} = 0.23919 + (10.5051)^2 \Big[0.004310824(-\lambda)^4 - 0.008621148(-\lambda)^3$$

$$+ 0.056220149(-\lambda)^2 - 0.010873779(-\lambda) + 0.009121777 \Big]$$

$$= \boxed{\underline{1.2024}} \qquad (0.9502350).$$

$$\Theta = f \quad \underline{0.2653712}$$

Check !!!

$$\frac{\sigma R}{Et} = \frac{2}{3(1-\nu^2)}\gamma \frac{2\rho-2}{3\rho-2} + \frac{1}{\gamma(3\rho-2)}\Big\{(\gamma\eta)^2(0.333125\rho^3 - 0.85875\rho^2 + 0.6975\rho - 0.125) + (\gamma\eta)(0.9375\rho^2 - 1.250\rho) + (0.25\rho - 0.5)\Big\}$$

$$= \frac{0.2}{3(1-\nu^2)}\,\frac{0.10102}{1.15153} + \frac{10}{1.15153}\Big\{0.333125\rho^3 + 0.07875\rho^2 + 0.1975\rho - 0.625\Big\}$$

$$= \underline{\underline{0.48907}}$$

O.K.

$\left(\frac{\varepsilon R}{t}\right) = 0.9748 \qquad \Phi = +0.124455$

$\eta = 10 \qquad \boxed{\gamma = 0.144 \qquad \gamma\eta = 1.44} \qquad \rho = 10.7961, \quad \lambda = 0.0737316$

$$0.199584\rho^3 + 0.399168\rho^2 + 1.220163\rho - 2.033710 = 0$$

$$F(\rho) = \rho^3 + 2.00000\rho^2 + 6.113631\rho - 10.18974 = 0$$

$$F'(\rho) = 3\rho^2 + 4.0000\rho + 6.113631$$

$F(1) = -1.07611 \qquad F'(1) = 13.113631$

$F(1.082) = +0.03338 \qquad F'(1.082) = 13.954$

$.00239$

1.07961

$F(1.07961) = +0.00006$, o.K.

$$\rho^2 + 3.07961\rho + 9.43841 = 0$$

No more real Root

$\gamma^2 = 0.020736$ $\quad (\gamma\xi) = 1.5546384$

$(\gamma\xi)^2 = 2.4169006$

$$\Xi = 0.0461446 + (10.7951)^2 \{ 0.009441018(-\lambda)^4 - 0.018882036(-\lambda)^3$$

$$+ 0.085829781(-\lambda)^2 - 0.015660708(-\lambda) + 0.006603078 \}$$

$$= \underline{0.7347181} \qquad (0.847512)$$

$$\Theta = -\ 0.1503705$$

$\rho = 1.07961$, $\rho^2 = 1.16555$, $\rho^3 = 1.25834$

$$\frac{\sigma R}{Et} = 0.105495 + 6.944444 \left\{ \frac{0.296784\rho^3 - 0.273168\rho^2 + 0.368800\rho - 0.2592}{-0.92039} + 0.25 \right\}$$

$$= 0.105495 + 0.109257 = \underline{\underline{0.2148}}$$

Check

$$\frac{\sigma R}{Et} = \frac{0.288}{2.73} \frac{0.15922}{1.23883} + \frac{6.94444}{1.23883} \left\{ 0.690768\rho^3 - 0.430704\rho^2 + 0.396336\rho - 0.7592 \right\}$$

$$= 0.0135587 + 0.201254 = \underline{\underline{0.214813}}$$

$\boxed{\frac{\varepsilon R}{t}} = 0.0.92999$ $\Phi = +0.167600$

$\beta = 0.144$	$n = 15$,	$(n\beta) = 2.16$

$\varsigma = 17.0025$, $\lambda = \underline{\underline{0.1177768}}$

$$0.449064\rho^3 + 0.898128\rho^2 + 0.0264230\rho - 1.837870 = 0$$

$$F(\rho) = \rho^3 + 2.000000\rho^2 + 0.0588402\rho - 4.092668 = 0$$

$$F'(\rho) = 3\rho^2 + 4.000\rho + 0.0588402$$

$F(1.134) = +0.004243$ $F'(1.134) = 8.4527$

$$\frac{50}{1.13350}$$

$F(1.13350) = 0$, $\rho^2 + 3.13350\rho + 3.61066 = 0$ No more real root.

$\gamma^2 = 0.020736$, $(\gamma\xi) = 2.4483600$

$(\gamma\xi)^2 = 5.9944667$

$$\xi = 0.3350094 + (17.0025)^2 \{ 0.023415886 (-\lambda)^4 - 0.046831771 (-\lambda)^3$$

$$+ 0.113809542 (-\lambda)^2 - 0.044754594 (-\lambda) + 0.007769092 \}$$

$$(5.1061960)$$

$$= \underline{1.6932169}$$

$$\Theta = - 0.668432$$

$\rho = 1.13350$; $\rho^2 = 1.28482$, $\rho^3 = 1.45634$

$\gamma = 0.144$ $(\eta\gamma) = 2.16$

$$\frac{\sigma R}{Et} = \frac{0.288}{2.73}\,\frac{0.26700}{1.40050} + \frac{6.944444}{1.40050}\Big\{1.554228\rho^3 - 1.981584\rho^2 + 1.304256\rho - 1.083200\Big\}$$

$$= 0.0201122 + 0.558729 = \underline{\underline{0.5788}}$$

$\left(\frac{\varepsilon R}{t}\right) = \underline{\underline{2.2598}}$ $\Phi = -0.461363$

$\gamma = 0.121$ $\eta = 15.$ $(\eta\gamma) = 1.815$ $\xi = 16.56975$

$$0.3170692\rho^3 + 0.6341383\rho^2 + 0.7433961\rho - 2.02239875 = 0$$

$\lambda = \underline{\underline{0.0947359}}$

$$F(\rho) = \rho^3 + 2.00000\rho^2 + 2.344586\rho - 6.378414 = 0$$

$$F'(\rho) = 3\rho^2 + 4.0000\rho + 2.344586$$

$F(1.09) = -0.1516$ (1476) $F'(\rho) = 10.219$

$F(1.10470) = +0.000508$ (000049) $F'(\rho) = 10.424$

$F(1.10465) = 0$ $\rho = 1.10465$, $\rho^2 = 1.22025$, $\rho^3 = 1.34795$

$$\frac{\sigma R}{Et} = \frac{0.242}{2.73}\,\frac{0.20930}{1.31395} + \frac{8.264463}{1.31395}\Big\{1.0973879\rho^3 - 1.127353\rho^2 + 0.778972\rho - 0.911778\Big\}$$

$$= 0.014120 + 0.32887 = \underline{\underline{0.3430}}$$

$\left(\frac{\varepsilon R}{t}\right) = 1.7225$ (2.9659724) $\Phi = +0.078351$

$\gamma^2 = 0.014641, \quad (\gamma\zeta) = 2.0049398$

$(\gamma\zeta)^2 = 4.0197836$

$$\mathcal{E} = 0.117649 + (16.56975)^2 \{ 0.0157022280(-\lambda)^4 - 0.031404559(-\lambda)^3$$

$$+ 0.120209472(-\lambda)^2 - 0.026189231(-\lambda) + 0.005872186 \}$$

$$= \underline{1.3379312}$$

$$\Theta = -\underline{0.5489065}$$

$\gamma = 0.100, \quad \eta = 15 \quad (\eta\gamma) = 1.5$ $\xi = 16.21518$

$0.2165625\,\rho^3 + 0.438125\,\rho^2 + 1.158576\,\rho - 2.032152 = 0$ | $\lambda = -0.0749409$

$F(\rho) = \rho^3 + 2.00000\,\rho^2 + 5.349846\,\rho - 9.383674 = 0$

$F'(\rho) = 3\rho^2 + 4.00000\,\rho + 5.349846$

$F(1.08) = -0.013328 \qquad F'(\rho) = 13.169$

001012

$F(1.081012) = 0$

$\rho^2 + 3.081012\rho + 8.680457 = 0$ No more real root !!!

$\rho = 1.081012, \quad \rho^2 = 1.168587, \quad \rho^3 = 1.263257$

$$\frac{\sigma R}{Et} = \frac{0.2}{2.73}\,\frac{0.162024}{1.243036} + \frac{10}{1.243036}\left\{0.74953125\,\rho^3 - 0.5259375\,\rho^2 + 0.444375\,\rho - 0.78125\right\}$$

$= 0.00954908 + 0.25238 = \underline{\underline{0.2619}}$

$\left(\frac{\varepsilon R}{t}\right) = 1.3805 \quad (1.9057603) \qquad \gamma^2 = 0.01, \quad (\gamma\xi) = 1.621518$

$(\gamma\xi)^2 = 2.6293206$

$\mathcal{E} = 0.0685916 + (16.21518)^2\{0.01027074(-\lambda)^4 - 0.02054156(-\lambda)^3$

$+ 0.08947623(-\lambda)^2 - 0.01515393(-\lambda) + 0.00591092\}$

$= 1.4700144 \qquad \Theta = -\underline{0.2443597} \qquad \Phi = +0.358467$

$\gamma = 0.121, \quad \eta = 10, \quad (\eta\gamma) = 1.21$ $\quad \xi = 10.63807\ \underline{648}$

$(\eta\gamma)^2 = 1.4641 \quad \lambda = -0.0599799$

$$0.140919625\,\rho^3 + 0.28183925\,\rho^2 + 1.3446905\,\rho - 1.9190950 = 0$$

$$F(\rho) = \rho^3 + 2.000000\,\rho^2 + 9.542253\,\rho - 13.618365 = 0$$

$$F'(\rho) = 3\rho^2 + 4.000000\,\rho + 9.542253$$

$F(1.062) = -0.03103 \qquad F'(1.062) = 17.173$

$.001807$

$F(1.063807) = $ O.K.

$$\rho = 1.063807, \quad \rho^2 = 1.131685, \quad \rho^3 = 1.203894$$

$$\frac{\sigma R}{Et} = \frac{0.242}{2.73}\,\frac{0.127614}{1.191421} + \frac{8.264463}{1.191421}\Big\{0.4877283\rho^3 - 0.1229209\rho^2 + 0.2587097\rho - 0.6830125\Big\}$$

$$= 0.00949480 + 0.279340 = \underline{\underline{0.2888}}$$

(0.7781298)

$$\frac{\varepsilon R}{t} = 0.8824\ldots, \qquad \gamma^2 = 0.014641, \quad (\gamma\xi) = 1.2872065$$

$$(\gamma\xi)^2 = 1.6569006$$

$$\mathcal{E} = 0.0834054 + (10.63807)^2\Big\{0.006472268\,(-\lambda)^4 - 0.012944536\,(-\lambda)^3 + 0.068706006\,(-\lambda)^2 - 0.011952234\,(-\lambda) + 0.007591422\Big\}$$

$$= \underline{0.8890517}$$

$$\Theta = +\,\underline{0.1418157} \qquad \Phi = +\,0.129689$$

$\gamma = 0.100\ ,\quad \eta = 12.50$

$(\eta\gamma) = 1.250$

$(\eta\gamma)^2 = 1.5625$

$\xi = 13.311375$ ~~649~~

$$0.150390625\rho^3 + 0.30078125\rho^2 + 1.3276385\rho - 1.936527 = 0$$

$$F(\rho) = \rho^3 + 2.00000\rho^2 + 8.827934\rho - 12.876647 = 0$$

$\lambda = -0.0609539$

$$F'(\rho) = 3\rho^2 + 4.00000\rho + 8.827934$$

$F(1.062) = -0.047923 \qquad F'(1.062) = 16.459$

00291

$F(1.06491) = +0.000018, \quad$ O.K.

$\rho = 1.064910, \quad \rho^2 = 1.134033, \quad \rho^3 = 1.207643$

$$\frac{\sigma R}{Et} = \frac{0.2}{2.73}\,\frac{0.129820}{1.194730} + \frac{10}{1.194730}\left\{0.52050781\rho^3 - 0.16992195\rho^2 + 0.27734381\rho - 0.6953125\right\}$$

$$= 0.007960 + 0.30069 = \underline{0.30865}$$

$$\left(\frac{\varepsilon R}{t}\right) = \underline{1.0759} \quad (1.1575608)$$

$\gamma^2 = 0.01, \quad (\gamma\xi) = 1.3311325$

$(\gamma\xi)^2 = 1.7719270$

$$\zeta = 0.0952148 + (13.311375)^2\left\{0.006921590(-\lambda)^4 - 0.013843180(-\lambda)^3 + 0.070788222(-\lambda)^2 - 0.011869074(-\lambda) + 0.007119617\right\}$$

$$= \underline{1.2747}$$

$$\Phi = +0.305273$$

$$\Delta\Theta = +\underline{0.1011949}$$

$\boxed{\gamma = 0.121, \quad \eta = 12.5}$ $\quad (\eta\gamma) = 1.5125$ $\quad$ 650

$\gamma^2 = 0.014641 \quad (\gamma\eta)^2 = 2.28765625 \quad \xi = 13.53684$

$$0.2201869\rho^3 + 0.4403738\rho^2 + 1.14973305\rho - 2.0412067 = 0 \quad \lambda = -0.0765939$$

$$F(\rho) = \rho^3 + 2.000000\rho^2 + 5.221623\rho - 9.270337 = 0$$

$$F'(\rho) = 3\rho^2 + 4.000000\rho + 5.221623$$

$$F(1.0829) = -0.000610 \qquad F'(1.0829) = 13.07124$$

000047

$$F(1.082947) = 0$$

$$\rho = 1.082947, \quad \rho^2 = 1.172774, \quad \rho^3 = 1.270052$$

$$\frac{\sigma R}{Et} = \frac{0.242}{2.73}\frac{0.165894}{1.248841} + \frac{8.2644628}{1.248841}\{0.7620754\rho^3 - 0.5465560\rho^2 + 0.4550152\rho - 0.7859570\}$$

$$= 0.0117754 + 0.222945 = \underline{0.23472}$$

$$\frac{\varepsilon R}{t} = \underline{1.1761} \quad (1.3832112)$$

$(\xi\gamma) = 1.6329571$

$(\xi\gamma)^2 = 2.6629051$

$$\varepsilon = 0.0550935 + (13.53684)^2\{0.01046098(-\lambda)^4 - 0.02091019(-\lambda)^3 + 0.08986 3022(-\lambda)^2 - 0.01660 6881(-\lambda) + 0.0060 81249\}$$

$$= \underline{1.031318}$$

$$\Theta = -\underline{0.2544031} \qquad \Phi = +0.239605$$

$\boxed{\gamma = 0.144 \qquad \eta = 12.5}$ $\quad (\eta\gamma) = 1.8000$ $\quad \xi = 13.80915$ $\quad$ 651

$\gamma^2 = 0.020736$ $\quad (\eta\gamma)^2 = 3.24000$

$\lambda = 0.0948031$

$$0.3118500\rho^3 + 0.62370000\rho^2 + 0.7729912\rho - 2.0335824 = 0$$

$$F(\rho) = \rho^3 + 2.000023\rho^2 + 2.478728\rho - 6.527441 = 0$$

$$F'(\rho) = 3\rho^2 + 4.000000\rho + 2.478728$$

$$F(1.1047) = -0.000332 \qquad F'(1.1047) = 10.5586$$

0000316

$$\rho = 1.104732, \quad \rho^2 = 1.220432, \quad \rho^3 = 1.348250$$

$$\frac{\sigma R}{Et} = \frac{0.288}{2.73}\,\frac{0.209464}{1.314196} + \frac{6.944444}{1.314196}\Big\{1.079325\rho^3 - 1.094855\rho^2 + 0.759900\rho - 0.9050000\Big\}$$

$$= 0.016814 + 0.282281 = \underline{\underline{0.29949}}$$

$$(2.0718724)$$

$$\left(\frac{\varepsilon R}{t}\right) = \underline{\underline{1.4394}} \qquad (\gamma\xi) = 1.9885176$$

$$(\gamma\xi)^2 = 3.9542022$$

$$\mathcal{E} = 0.0896943 + (13.80915)^2\Big\{0.015446102(-\lambda)^4 - 0.03092205(-\lambda)^3 + 0.119338152(-\lambda)^2 - 0.0262155(-\lambda) + 0.006130651\Big\}$$

$$= \underline{\underline{0.98458}}$$

$$\angle\,\Theta = \underline{-0.5248873} \qquad \Phi = +0.061204$$

$\gamma = 0.100 \quad \eta = 17.5$ $\quad (\eta\gamma) = 1.750 \quad (\eta\gamma)^2 = 3.0625 \quad \xi = 19.2293325 \quad \lambda = -0.0899320$

$$0.294765625\,\rho^3 + 0.58953125\,\rho^2 + 0.8451385\,\rho - 2.0315270 = 0$$

$$F(\rho) = \rho^3 + 2.000000\,\rho^2 + 2.867154\,\rho - 6.892008 = 0$$

$$F'(\rho) = 3\rho^2 + 4.000000\,\rho + 2.867154$$

$$F(1.0988) = -0.000208 \qquad F'(1.0988) = 10.48$$

000019

$$\rho = 1.098819, \quad \rho^2 = 1.207403, \quad \rho^3 = 1.326717$$

$$\frac{\sigma R}{Et} = 0.007326007 \times \frac{0.197638}{1.296457} + \frac{10}{1.296457}\left\{1.0201953\,\rho^3 - 0.9892969\,\rho^2 + 0.6985938\,\rho - 0.8828125\right\}$$

$$= 0.0111681 + 0.338198 = \underline{0.34938}$$

$$\frac{\varepsilon R}{t} = \underline{1.8935} \qquad (3.5153473)$$

$$(\gamma\xi) = 1.92293325$$

$$(\gamma\xi)^2 = 3.6976725$$

$$\mathcal{C} = 0.1220664 + (19.2293325)^2\left\{0.014444033\,(-\lambda)^4 - 0.028888066\,(-\lambda)^3 + 0.1127634356\,(-\lambda)^2 - 0.023204843\,(-\lambda) + 0.005584763\right\}$$

$$= \underline{1.74529}$$

$$\Delta\Theta = \underline{-0.5132152} \qquad \Phi = +0.211094$$

$$K = -4\left(\frac{\sigma}{E}\right)^2 - m^2 \frac{\sigma}{E}\left[\frac{3}{8} f_1^2 + \frac{1}{2} f_1 f_2 + \frac{1}{9} f_2^2\right]$$

$$\delta_1 = \frac{1}{8}\left[m^4\left(\frac{533}{3200} f_1^4 + \frac{229}{400} f_1^3 f_2 + \frac{279}{400} f_1^2 f_2^2 + \frac{1}{4} f_1 f_2^3 + \frac{1}{8} f_2^4\right) - m^2\left(\frac{5}{8} f_1^3 + \frac{5}{4} f_1^2 f_2\right) + \left(\frac{3}{4} f_1^2 + f_1 f_2 + f_2^2\right) + \left(\lambda + \frac{3}{2}\right)\right]$$

$$\delta_2 = \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \left[f_1^2 + 2 f_1 f_2 + 2 f_2^2\right]$$

$$\frac{\sigma R}{Et}\gamma\left(\frac{1}{2}\lambda + \frac{3}{4}\right) = \frac{1}{8}\left[(\gamma\xi)^2\left(\frac{1}{4}\lambda^3 + \frac{279}{200}\lambda^2 + \frac{687}{400}\lambda + \frac{533}{800}\right) - (\gamma\xi)\left(\frac{5}{2}\lambda + \frac{15}{8}\right) + \frac{1}{3(1-\nu^2)}\gamma^2(\varsigma + 1)\right.$$

$$\frac{\sigma R}{Et}\gamma\left(\lambda + \frac{1}{2}\right) = \frac{1}{8}\left[(\gamma\xi)^2\left(\frac{1}{2}\lambda^3 + \frac{3}{4}\lambda^2 + \frac{279}{200}\lambda + \frac{229}{400}\right) - (\gamma\xi)\left(\quad + \frac{5}{4}\right) + (2\lambda + 1)\right] + \frac{1}{3(1-\nu^2)}\gamma^2(2\lambda + 1)$$

$$\frac{\sigma R}{Et}\gamma(2\lambda + 3) = (\gamma\xi)^2\left(\frac{1}{8}\lambda^3 + \frac{279}{400}\lambda^2 + \frac{687}{800}\lambda + \frac{533}{1600}\right) - (\gamma\xi)\left(\frac{5}{4}\lambda + \frac{15}{16}\right) + \left(\frac{\lambda}{2} + \frac{3}{4}\right) + \frac{2}{3(1-\nu^2)}\gamma^2(2\lambda + 2)$$

$$\frac{\sigma R}{Et}\gamma(2\lambda + 1) = (\gamma\xi)^2\left(\frac{1}{8}\lambda^3 + \frac{75}{400}\lambda^2 + \frac{279}{800}\lambda + \frac{229}{1600}\right) - (\gamma\xi)\left(0 + \frac{5}{16}\right) + \left(\frac{\lambda}{2} + \frac{1}{4}\right) + \frac{2}{3(1-\nu^2)}\gamma^2(2\lambda + 1)$$

$$\lambda = \frac{f_2}{f_1} \qquad \xi = \frac{f_1}{\frac{t}{R}}, \quad = \frac{\delta}{t} \quad !!!$$

$$\gamma = m^2 \frac{t}{R}$$

$$0 = (\gamma\xi)^2\left(\frac{77}{100}\lambda^3 + \frac{231}{200}\lambda^2 + \frac{77}{400}\lambda - \frac{77}{800}\right) - (\gamma\xi)\left(\frac{5}{2}\lambda^2 + \frac{5}{2}\lambda\right) - \frac{2}{3(1-\nu^2)}\gamma^2(2\lambda+1)$$

$$\left\{\frac{77}{100}(\gamma\xi)^2\right\}\lambda^3 + \left\{\frac{231}{200}(\gamma\xi)^2 - \frac{5}{2}(\gamma\xi)\right\}\lambda^2 + \left\{\frac{77}{400}(\gamma\xi)^2 - \frac{5}{2}(\gamma\xi) - \frac{4}{3(1-\nu^2)}\gamma^2\right\}\lambda - \left\{\frac{77}{800}(\gamma\xi)^2 + \frac{2}{3(1-\nu^2)}\gamma^2\right\} = 0$$

$$\frac{\sigma R}{E i} = \frac{2\gamma}{3(1-\nu^2)}\,\frac{2(\lambda+1)}{2\lambda+3} + \frac{1}{\gamma(2\lambda+3)}\Big\{0.1250000(\gamma\xi)^2\lambda^3 + 0.6975000(\gamma\xi)^2\lambda^2 + \left[0.1587500(\gamma\xi)^2 - 1.2500(\gamma\xi) + 0.5\right]\lambda + \left[0.3331250(\gamma\xi)^2 - 0.9375(\gamma\xi) + 0.75000\right]\Big\}$$

$\boxed{\gamma = 0.121, \quad \xi = 8,}$ $(\gamma\xi) = 0.968$

$(\gamma\xi)^2 = 0.937024$

$$0.7215085\,\lambda^3 - 1.3377373\,\lambda^2 - 2.2610749\,\lambda - 0.10091456 = 0$$

$$F(-\lambda) = \lambda^3 + 1.854084\,\lambda^2 - 3.133816\,\lambda + 0.1398661 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 3.908168\,\lambda - 3.133816$$

$F(0.04) = +0.017544$ $\quad F'(0.04) = 2.9727$

00590

$F(0.04590) = +0.0000268$ $\quad F'(0.04590) = 2.948$

91

$F(0.0459091) =$ O.K.

$$\lambda = -0.0459091, \quad \lambda^2 = +0.0021076, \quad \lambda^3 = -0.0000968$$

$$\frac{\sigma R}{Et} = \frac{0.242}{2.73}\,\frac{1.9081818}{2.9081818} + \frac{1}{0.351890}\left\{0.117128\,\lambda^3 + 0.153574\,\lambda^2 + 0.096670\,\lambda + 0.154646\right\}$$

$$= 0.058164 + 0.431004 = \underline{\underline{0.4892}}$$

$$\left(\frac{\varepsilon R}{t}\right) = \underline{0.8310} \quad (0.6905610)$$

$$\mathcal{E} = 0.2393166 + 64\left\{0.003660250\,(-\lambda)^4 - 0.007320500\,(-\lambda)^3 + 0.053014946\,(-\lambda)^2 - 0.011542196\,(-\lambda) + 0.010078909\right\} = \underline{0.85756}$$

$\Theta = +\underline{0.2418309}$ $\quad \Phi = +0.022255$

$\gamma = 0.144, \quad \xi = 8,$

$(\gamma\xi) = 1.152$

$(\gamma\xi)^2 = 1.327104$

$$1.0218701\,\lambda^3 + 1.3471949\,\lambda^2 - 2.6549149\,\lambda + 0.1429250 = 0$$

$$F(-\lambda) = \lambda^3 + 1.318362\,\lambda^2 - 2.598094\,\lambda + 0.1398661 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 2.636724\,\lambda - 2.598094$$

$F(0.054) = 0.0035708$ (146) $\qquad F'(0.054) = 2.447$

$F(0.05546) = +0.0000014,$ (0000006) $\qquad F'(0.05546) = 2.443$

$$\lambda = -0.0554606, \quad \lambda^2 = +0.0030759, \quad \lambda^3 = -0.0001706$$

$$\frac{\sigma R}{Et} = \frac{0.288}{2.73}\,\frac{1.8890788}{2.8890788} + \frac{1}{0.4160273}\left\{0.165888\,\lambda^3 + 0.92565504\,\lambda^2 + 0.1996506\,\lambda + 0.1120915\right\}$$

$$= 0.068980 + 0.24959 = \underline{0.31857}$$

$$\frac{\varepsilon R}{t} = \underline{0.7204} \quad (0.5179212)$$

$$\mathcal{E} = 0.1014868 + 64\left\{0.005184000\,(-\lambda)^4 - 0.010368000\,(-\lambda)^3 + 0.062075621\,(-\lambda)^2 - 0.011891621\,(-\lambda) + 0.008794632\right\}$$

$$= \underline{0.134244}$$

$\Theta = \underline{+0.2221061} \qquad \Phi = +0.087624$

$\boxed{\gamma = 0.144 \quad . \quad \xi = 6,}$ $\quad (\gamma\xi) = 0.864$

$(\gamma\xi)^2 = 0.746496$

$$0.57480192\lambda^3 + 1.2977971\lambda^2 - 2.0466819\lambda + 0.08703144 = 0$$

$$F(-\lambda) = \lambda^3 + 2.257817\lambda^2 - 3.560674\lambda + 0.1514112 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 4.515634\lambda - 3.560674$$

$$F(0.0435) = +0.0008765 \qquad F'(0.0435) = -3.3586$$

261

$$F(0.043761) = \text{o.k.}$$

$$\lambda = -0.0437610, \quad \lambda^2 = +0.0019150, \quad \lambda^3 = -0.0000838$$

$$\frac{\sigma R}{Et} = \frac{0.288}{2.73}\,\frac{1.912478}{2.912478} + \frac{1}{0.4193968}\left\{0.0933120\lambda^3 + 0.5206810\lambda^2 + 0.0610534\lambda + 0.1886765\right\}$$

$$= 0.06927 + 0.44586 = 0.51513$$

$$\frac{\varepsilon R}{t} = \underline{0.7446} \quad (0.5544292)$$

$$\mathcal{E} = 0.2653589 + 36\left\{0.002916000(-\lambda)^4 - 0.005832000(-\lambda)^3 + 0.049420181(-\lambda)^2 - 0.012754181(-\lambda) + 0.011392521\right\}$$

$$= \underline{0.6589661}$$

$$\Theta = \underline{+0.1885496} \qquad \Phi = -0.054082$$

$\gamma = 0.169, \quad \xi = 5,$ $\quad (\gamma\xi) = 0.845$

$(\gamma\xi)^2 = 0.714025$

$\frac{2\gamma}{3(1-\nu^2)} = 0.1238095, \quad \frac{2\gamma^2}{3(1-\nu^2)} = 0.0209238$

$0.54979925\lambda^3 + 1.2878011\lambda^2 - 2.0168878\lambda + 0.089648706 = 0$

$F(-\lambda) = \lambda^3 + 2.3423115\lambda^2 - 3.6684077\lambda + 0.1630572 = 0$

$F'(-\lambda) = 3\lambda^2 + 4.6846230\lambda - 3.6684077$

$F(0.0457) = + 0.0003983 \qquad F'(0.0457) = -3.4480$

0001155

$F(0.0458155) = + 0.0 -$

$\lambda = -0.0458155, \quad \lambda^2 = +0.0020991, \quad \lambda^3 = -0.0000962$

$$\frac{\sigma R}{Et} = 0.1238095\,\frac{1.9083690}{2.9083690} + \frac{1}{0.4915144}\{0.0892531\lambda^3 + 0.4980324\lambda^2 + 0.0569180\lambda + 0.1956721\}$$

$= 0.08124 + 0.39490 = \underline{\underline{0.47614}}$

(0.4390388)

$\left(\frac{\varepsilon R}{t}\right) = \underline{\underline{0.6626}} \qquad \gamma^2 = 0.028561$

$\xi = 0.2267093 + 25\{0.002789160(-\lambda)^4 - 0.105578320(-\lambda)^3 + 0.049428990(-\lambda)^2$

$- 0.013632017(-\lambda) + 0.011957888\}$

$= \underline{0.5126213}$ $\qquad \Theta = +\underline{0.1676036} \qquad \Phi = -0.059178$

$\boxed{\gamma = 0.169,\ \xi = 7.}$ $(\gamma\xi) = 1.183$

$(\gamma\xi)^2 = 1.399489$

$$1.07760653\,\lambda^3 + 1.3410902\,\lambda^2 - 2.7299460\,\lambda + 0.1556246 = 0$$

$$F(-\lambda) = \lambda^3 + 1.2445082\,\lambda^2 - 2.5333421\,\lambda + 0.1444169$$

$$F'(-\lambda) = 3\lambda^2 + 2.4890164\,\lambda - 2.5333421$$

$F(0.0590) = -\ 0.0005128$ $F'(0.0590) = 2.376$

$\underline{0002158}$

$F(0.0587842) = $ o.k.

$\lambda = -0.0587842,\quad \lambda^2 = +0.0034556,\quad \lambda^3 = -0.0002031$

$$\frac{\sigma R}{Et} = 0.1238095 \times \frac{1.8824316}{2.8824316} + \frac{1}{0.4871309}\left\{0.1749361\,\lambda^3 + 0.9761436\,\lambda^2 + 0.2230612\,\lambda + 0.1071423\right\}$$

$= 0.08086 + 0.19988 = \underline{0.28074}$

$$\left(\frac{\varepsilon R}{t}\right) = \underline{\underline{0.6403}} \quad (0.4099841)$$

$\gamma^2 = 0.028561$

$$\varepsilon = 0.0788149 + 49\left\{0.005466754(-\lambda)^4 - 0.010933508(-\lambda)^3 + 0.064369963(-\lambda)^2 - 0.012692272(-\lambda) + 0.008924220\right\}$$

$= \underline{0.490331}$

$\Phi = +\ 0.065410$

$\Theta = +\ \underline{0.1959878}$

$\boxed{\gamma = 0.169 \quad \xi = 9}$

$\sqrt{\xi} = 1.521$

$\sqrt{\xi}^2 = 2.313441$

$$1.7813496\,\lambda^3 + 1.1304756\,\lambda^2 - 3.3991102\,\lambda + 0.2435925 = 0$$

$$F(-\lambda) = \lambda^3 + 0.6346175\,\lambda^2 - 1.9081657\,\lambda + 0.1367460$$

$$F'(-\lambda) = 3\lambda^2 + 1.2692350\,\lambda - 1.9081657$$

$F(0.073) = +0.001221$ $\qquad F'(0.073) = 1.7995$

$F(0.0736786) =$ O.K.

$$\lambda = -0.0736786, \quad \lambda^2 = +0.0054285, \quad \lambda^3 = -0.0004000$$

$$\frac{\sigma R}{Et} = 0.1238095 \times \frac{1.8526428}{2.8526428} + \frac{1}{0.4820966}\left\{0.2891801\,\lambda^3 + 1.6136251\,\lambda^2 + 0.5854175\,\lambda + 0.0947275\right\}$$

$$= 0.08041 + 0.12495 = \underline{\underline{0.20536}}$$

$\gamma^2 = 0.028561$

(0.6218900)

$$\left(\frac{\varepsilon R}{t}\right) = \underline{\underline{0.7886}}$$

$$\zeta = 0.0421727 + 81\left\{0.009036179\,(-\lambda)^4 - 0.018073758\,(-\lambda)^3 + 0.084276701\,(\lambda)^2 - 0.015840319\,(-\lambda) + 0.007079050\right\}$$

$$= \underline{0.5575996}$$

$$\Phi = +0.116853$$

$$\Theta = -\underline{0.1033791}$$

$\boxed{\gamma = 0.169 \quad \xi = 11}$ $(\gamma\xi) = 1.859$

$(\gamma\xi)^2 = 3.455881$

$$2.6610284\lambda^3 + 0.6559574\lambda^2 - 4.0240905\lambda + 0.3535523 = 0$$

$$F(-\lambda) = \lambda^3 + 0.2465052\lambda^2 - 1.5122315\lambda + 0.1328630 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 0.4930104\lambda - 1.5122315$$

$$F(0.0894) = +0.0003542, \qquad F'(0.0894) = 1.4442$$

2453

$$F(0.0891453) = 0$$

$$\lambda = -0.0896453, \quad \lambda^2 = +0.0080363, \quad \lambda^3 = -0.0007204$$

$$\frac{\sigma R}{Et} = 0.1238095 \times \frac{1.8707094}{2.8707094} + \frac{1}{0.4766999}\left\{0.4319851\lambda^3 + 2.4104770\lambda^2 + 1.1439876\lambda + 0.1584279\right\}$$

$$= 0.07992 + 0.15719 = \underline{\underline{0.23711}}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{1.0914}} \quad (1.1911540)$$

$$\gamma^2 = 0.028561$$

$$\zeta = 0.0562212 + 121\left\{0.013499535(-\lambda)^4 - 0.026999070(-\lambda)^3 + 0.109192882(-\lambda)^2 - 0.023076160(-\lambda) + 0.006424777\right\}$$

$$= \underline{0.6872404}$$

$$\Theta = -\underline{0.4230466} \qquad \Phi = +0.084838$$

$\gamma = 0.225 \quad \xi = 4,$ $(\gamma\xi) = 0.900$

$(\gamma\xi)^2 = 0.810$

$\frac{2\gamma}{3(1-\nu^2)} = 0.1648352, \quad \frac{2\gamma^2}{3(1-\nu^2)} = 0.0370879$

$0.6237\,\lambda^3 + 1.31445\,\lambda^2 - 2.1682508\,\lambda + 0.1150504 = 0$

$F(-\lambda) = \lambda^3 + 2.1075036\,\lambda^2 - 3.4764322\,\lambda + 0.1844643$

$F'(-\lambda) = 3\lambda^2 + 4.2150072\,\lambda - 3.4764322$

$F(0.0549) = 0.0001257$ $\quad F'(0.0549) = 3.2360$

0000388

$F(0.0549389) =$ o.k.

$\lambda = -0.0549389, \quad \lambda^2 = +0.0030183, \quad \lambda^3 = -0.0001658$

$\frac{\sigma R}{Et} = 0.1648352 \times \frac{1.8901222}{2.8901222} + \frac{1}{0.6502775}\{0.10125\,\lambda^3 + 0.564975\,\lambda^2 + 0.0705875\,\lambda + 0.1760813\}$

$= 0.10780 + 0.26741 = \underline{0.37521}$

(0.2733433)

$\left(\frac{\varepsilon R}{t}\right) = \underline{0.5323}$ $\quad \gamma^2 = 0.050625$

$\xi = 0.1407825 + 16\{0.003164063(-\lambda)^4 - 0.006328125(-\lambda)^3 + 0.05354058(-\lambda)^2 - 0.015221145(-\lambda) + 0.012393484\}$

$= \underline{0.3282678}$ $\quad \Theta = +\underline{0.1585515}$ $\quad \Phi = -0.035590$

$\gamma = 0.225 \quad \xi = 6$

$(\gamma\xi) = 1.350$

$(\gamma\xi)^2 = 1.8225$

$$1.4033250\,\lambda^3 + 1.2700125\,\lambda^2 - 3.0963446\lambda + 0.2\,1250353 = 0$$

$$F(-\lambda) = \lambda^3 + 0.9050024\,\lambda^2 - 2.2064344\,\lambda + 0.1514286 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 1.8100048\,\lambda - 2.2064344$$

$F(0.0707\ \ 1506) = +\ 0.0003107 \qquad F'(0.0707) = 2.06347$

$F(0.0708506) = $ O.K.

$\lambda = -0.0708506, \quad \lambda^2 = +0.0050198, \quad \lambda^3 = -0.0003557$

$$\frac{\sigma R}{Et} = 0.1648352 \times \frac{1.8582988}{2.8582988} + \frac{1}{0.6431172}\left\{0.2278125\,\lambda^3 + 1.2711938\,\lambda^2 + 0.3775719\lambda + 0.0914953\right\}$$

$$= 0.10717 + 0.11047 = \underline{0.21764}$$

(0.3180960)

$$\left(\frac{\varepsilon R}{t}\right) = \underline{0.5640} \qquad \gamma^2 = 0.050625$$

$$\mathcal{E} = 0.0473672 + 36\left\{0.007119141(-\lambda)^4 - 0.01423828\,1(-\lambda)^3 + 0.07561\,0794(-\lambda)^2 - 0.01575\,7278(-\lambda) + 0.00887456\,4\right\}$$

$$= \underline{0.3401487}$$

$\Theta = +0.0693272 \qquad \Phi = +0.047326$

$\nu = 0.225, \quad \xi = 7.5$

$(\gamma\xi) = 1.6875$

$(\gamma\xi)^2 = 2.84765625$

$$2.1926953\,\lambda^3 + 0.9297070\,\lambda^2 - 3.7447520\,\lambda + 0.3111748 = 0$$

$$F(-\lambda) = \lambda^3 + 0.4240019\,\lambda^2 - 1.7078305\,\lambda + 0.1419143 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 0.8480038\,\lambda - 1.7078305$$

$F(0.0852) = +\,0.0001035$ (641) $\qquad F'(0.0852) = 1.6138$

$F(0.0852641) =$ O.K.

$$\lambda = -\,0.0852641, \quad \lambda^2 = +0.0072700, \quad \lambda^3 = -\,0.0006199$$

$$\frac{\sigma R}{Et} = 0.1648352 \times \frac{1.8294718}{2.8294718} + \frac{1}{0.6366312}\left\{0.3559570\,\lambda^3 + 1.9862402\,\lambda^2 + 0.8360498\,\lambda + 0.1165942\right\}$$

$$= 0.10658 + 0.09351 = \underline{0.20009}$$

(0.5353849)

$$\frac{\varepsilon R}{t} = \underline{0.7317}$$

$$\zeta = 0.040036 + 56.25\left\{0.011123657(-\lambda)^4 - 0.022247314(-\lambda)^3 + 0.097955996(-\lambda)^2 - 0.020914370(-\lambda) + 0.007618776\right\}$$

$$= \underline{0.4071002}$$

$\Theta = -\,0.2386782 \qquad \Phi = +\,0.057394$

$\boxed{\gamma = 0.225, \quad \xi = 9,}$ $(\gamma\xi) = 2.025$ 665

$(\gamma\xi)^2 = 4.100625$

$$3.1574813\,\lambda^3 + 0.3262781\,\lambda^2 - 4.3473055\,\lambda + 0.4317731 = 0$$

$$F(-\lambda) = \lambda^3 + 0.1033349\,\lambda^2 - 1.3768269\,\lambda + 0.1367460$$

$$F'(-\lambda) = 3\lambda^2 + 0.2066698\,\lambda - 1.3768269$$

$F(0.10 \quad) = +0.0010966$ $\qquad F'(0.10) = -1.326$

0008270

$F(0.1008270) = $ o.k.

$$\lambda = -0.1008270, \quad \lambda^2 = +0.0101661, \quad \lambda^3 = -0.0010250$$

$$\frac{\sigma R}{Et} = 0.1648352 \times \frac{1.7983460}{2.7983460} + \frac{1}{0.6296279}\left\{0.5125781\,\lambda^3 + 2.8601859\,\lambda^2 + 1.4901617\,\lambda + 0.2175832\right\}$$

$$= 0.10593 + 0.15229 = \underline{0.25822}$$

(1.0184846)

$$\frac{\varepsilon R}{t} = \underline{1.0892} \qquad \gamma^2 = 0.050625$$

$$\xi = 0.0661726 + 81\left\{0.016018066(-\lambda)^4 - 0.032031133(-\lambda)^3 + 0.125266299(-\lambda)^2 - 0.030147171(-\lambda) + 0.007548791\right\}$$

$$= \underline{0.5325437}$$

$$\Theta = -\underline{0.4771215} \qquad \Phi = +0.005676$$

$\gamma = 0.400, \quad \xi = 1,$ $\quad (\gamma\xi) = 0.400$

$(\gamma\xi)^2 = 0.1600$

$$\frac{2\gamma}{3(1-\nu^2)} = 0.2930403, \quad \frac{2\gamma^2}{3(1-\nu^2)} = 0.1172161$$

$$0.1232\lambda^3 + 0.8152\lambda^2 - 1.2036322\lambda + 0.1326161 = 0$$

$$F(-\lambda) = \lambda^3 + 6.6168831\lambda^2 - 9.7697419\lambda + 1.0764294 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 13.2337662\lambda - 9.7697419$$

$F(0.120) = +0.0010715$ $\quad F'(0.120) = 8.138$

0001317

$F(0.1201317) =$ 0.K.

$$\lambda = -0.1201317, \quad \lambda^2 = 0.0144316, \quad \lambda^3 = -0.0017337$$

$$\frac{\sigma R}{Et} = 0.2930403 \times \frac{1.7597366}{2.7597366} + \frac{1}{1.1038946}\left\{0.02\lambda^3 + 0.1116\lambda^2 + 0.1374\lambda + 0.4283\right\}$$

$$= 0.18686 + 0.37446 = \underline{0.56132}$$

(0.3333908)

$$\frac{\varepsilon R}{t} = \underline{0.5774} \qquad \gamma^2 = 0.16000$$

$$\Sigma = 0.3150801 + \left\{0.000625(-\lambda)^4 - 0.00125(-\lambda)^3 + 0.04939514(-\lambda)^2 - 0.033139514(-\lambda) + 0.02348321\right\}$$

$$= \underline{0.335594} \qquad \Theta = \underline{+0.0066085} \qquad \Phi = -0.156309$$

$\boxed{\gamma = 0.400 \quad \xi = 2.5}$ $(\xi\gamma) = 1.000$ 667

$(\xi')^2 = 1.000$

$$0.77\lambda^3 + 1.345\lambda^2 - 2.5419322\lambda + 0.2134661 = 0$$

$$F(-\lambda) = \lambda^3 + 1.7467532\lambda^2 - 3.3012107\lambda + 0.2772287 = 0$$

$$F'(-\lambda) = 3\lambda^3 + 3.4935064\lambda - 3.3012107$$

$F(0.0883) = +0.0000395$ $F'(0.0883) = 2.969$

0000133

$F(0.0883133) = $ O.K.

$\lambda = -0.0883133$, $\lambda^2 = +0.0077992$, $\lambda^3 = -0.0006888$

$$\frac{\sigma R}{Et} = 0.2930403 \times \frac{1.8233734}{2.8233734} \times \frac{1}{1.1293494}\{0.125\lambda^3 + 0.6975\lambda^2 + 0.10875\lambda + 0.145625\}$$

$= 0.18925 + 0.12518 = \underline{0.31443}$

$\left(\frac{\varepsilon R}{t}\right) = \underline{0.419_0}$ (0.1755610)

$\xi = 0.0988662 + 6.25\{0.0039625(-\lambda)^4 - 0.0078125(-\lambda)^3$

$+ 0.067698889(-\lambda)^2 - 0.024730139(-\lambda) + 0.016437337\}$

$= \underline{0.1912174}$

$\Theta = +\underline{0.0891793}$ $\Phi = -0.03409$

$\gamma = 0.400 \quad \xi = 4$ $\quad (\gamma\xi) = 1.600$ $\quad$ 668

$(\gamma\xi)^2 = 2.560$

$$1.9712\lambda^3 + 1.0432\lambda^2 - 3.7416322\lambda + 0.3636161 = 0$$

$$F(-\lambda) = \lambda^3 + 0.5292208\lambda^2 - 1.8981495\lambda + 0.1844673 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 1.0584416\lambda - 1.8981495 = 0$$

$F(0.10) = +0.0009415$, $\quad F'(0.10) = 1.762$

$F(0.1005343) = 0.K.$

$\lambda = -0.1005343$, $\quad \lambda^2 = +0.0101071$, $\quad \lambda^3 = -0.0010161$

$$\frac{\sigma R}{Et} = 0.2930403 \times \frac{1.7989314}{2.7989314} + \frac{1}{1.1195726}\left\{0.32\lambda^3 + 1.7856\lambda^2 + 0.6984\lambda + 0.1028\right\}$$

$$= 0.18834 + 0.04494 = \underline{0.23328}$$

(0.2471084)

$$\left(\frac{\varepsilon R}{t}\right) = \underline{0.4971}$$

$$\varepsilon = 0.0544196 + 16\left\{0.01(-\lambda)^4 - 0.02(-\lambda)^3 + 0.101702014(-\lambda)^2 - 0.029202014(-\lambda) + 0.012838510\right\}$$

$= \underline{0.2290008}$

$\Theta = \underline{-0.07327797}$ $\quad \Phi = -0.001459$

$\gamma = 0.400, \quad \xi = 5.5$ $\quad (\gamma\xi) = 2.20$

$(\gamma\xi)^2 = 4.84$

$$3.7268\lambda^3 - 0.0902\lambda^2 - 4.8027322\lambda + 0.5830661 = 0$$

$$F(-\lambda) = \lambda^3 - 0.0242031\lambda^2 - 1.2887013\lambda + 0.1564522 = 0$$

$$F'(-\lambda) = 3\lambda^2 - 0.0484062\lambda - 1.2887013$$

$F(0.10) = 0.0243401, \quad F'(0.10) = 1.2905$

$F(0.12195) = 0.0007487 \quad F'(0.12195) = 1.2450$

0006014

$F(0.1225494) =$ o.k.

$\lambda = -0.1225494, \quad \lambda^2 = +0.0150184, \quad \lambda^3 = -0.0018405$

$$\frac{\sigma R}{Et} = 0.2930403 \times \frac{1.7549012}{2.7549012} + \frac{1}{1.1019605}\left\{0.605\lambda^3 + 3.3759\lambda^2 + 1.90635\lambda + 0.2998250\right\}$$

$= 0.18667 + 0.10508 = \underline{0.29175}$

(0.6046618)

$$\frac{\varepsilon R}{t} = \underline{0.7776}$$

$$\varepsilon = 0.0851181 + 30.25\left\{0.01890625(-\lambda)^4 - 0.03781250(-\lambda)^3 + 0.151393889(-\lambda)^2 - 0.046555139(-\lambda) + 0.012987340\right\}$$

$= \underline{0.3722048}$ $\quad \Theta = \underline{-0.3844414}$ $\quad \Phi = -0.040762$

$\gamma = 0.676, \quad \xi = 0,$ $\quad \xi\gamma = 0$

$(\xi\gamma)^2 = 0$

$\frac{2\gamma}{3(1-\nu^2)} = 0.4952381, \quad \frac{2\gamma^2}{3(1-\nu^2)} = 0.3347810$

$\lambda = -0.5000000$

$$\frac{\sigma R}{Et} = 0.4952381 \times \frac{1}{2} + \frac{1}{1.352}\left\{-0.25 + 0.75\right\}$$

$$= 0.2476191 + 0.36982 = \underline{\underline{0.61744}}$$

$$\left(\frac{\varepsilon R}{t}\right) = \underline{\underline{0.61744}}$$

$\Theta = 0$

$\gamma = 0.676 \quad \xi = 0.5$

$(\gamma\xi) = 0.338$

$(\gamma\xi)^2 = 0.114244$

$$0.01796788\lambda^3 + 0.7130482\lambda^2 - 1.4925700\lambda + 0.3457770 = 0$$

$$F(-\lambda) = \lambda^3 + 8.1057791\lambda^2 - 16.9672158\lambda + 3.9307188 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 16.2115582\lambda - 16.9672158$$

$F(0.266 \quad) = +0.0097930$ (0007860) $\qquad F'(0.266) = -12.443$

$F(0.2667860) = +0.0000186$ (0000015) $\qquad F'(0.2667860) = -12.428$

$$\lambda = -0.2667875, \quad \lambda^2 = 0.0711756, \quad \lambda^3 = -0.0189888$$

$$\frac{\sigma R}{Et} = 0.4952381 \times \frac{1.4664250}{2.4664250} + \frac{1}{1.6673033}\{0.0142805\lambda^3 + 0.0796852\lambda^2 + 0.1751070\lambda + 0.4711825\}$$

$$= 0.29445 + 0.25774 = \underline{0.55219}$$

(0.311364)

$$\frac{\varepsilon R}{t} = \underline{0.5580} \qquad \gamma^2 = 0.456976$$

$$\mathcal{E} = 0.3049138 + 0.25\{0.000446266(-\lambda)^4 - 0.000892531(-\lambda)^3 + 0.075547721(-\lambda)^2 - 0.061938390(-\lambda) + 0.038354398\}$$

$$= \underline{0.3117121}$$

$$\Theta = +0.00112 \qquad \Phi = -0.150261$$

$\boxed{\gamma = 0.676, \qquad \xi = 1,}$ $(\gamma\xi) = 0.676$

$(\gamma\xi)^2 = 0.456976$

$-0.3518715\lambda^3 + 1.1621927\lambda^2 - 2.2715941\lambda + 0.3787649 = 0$

$F(-\lambda) = \lambda^3 + 3.3028896\lambda^2 - 6.4557491\lambda + 1.0764296 = 0$

$F'(-\lambda) = 3\lambda^2 + 6.6057792\lambda - 6.4557491$

$F(0.171) = 0.0740765$ $\qquad F'(0.171) = 5.238$

0.01414

$F(0.18514) = 0.0007708$ $\qquad F'(0.18514) = 5.1300$

0001503

$F(0.1852903) = 0.k.$

$\lambda = -0.1852903, \quad \lambda^2 = +0.0343325 \quad \lambda^3 = -0.0063615$

$$\frac{OR}{Et} = 0.4952381 \times \frac{1.6294194}{2.6294194} + \frac{1}{1.7774875}\Big\{0.0571220\lambda^3 + 0.3187408\lambda^2 + 0.0474281\lambda + 0.2684801\Big\}$$

$= 0.30689 + 0.15205 = \underline{0.45894}$

$\gamma^2 = 0.456976$

(0.2344496)

$$\frac{ER}{t} = \underline{0.4842}$$

$$\mathcal{E} = 0.2106259 + \Big\{0.001785063(-\lambda)^4 - 0.003570125(-\lambda)^3 + 0.08305826(-\lambda)^2$$

$$- 0.054866955(-\lambda) + 0.033536783\Big\}$$

$= \underline{0.24246}$

$\Theta = +\ \underline{0.03417} \qquad \Phi = -0.100989$

$\gamma = 0.676, \quad \xi = 0.75,$ $\quad (\gamma\xi) = 0.507$

$(\gamma\xi)^2 = 0.257049$

$$0.19792773\lambda^3 + 0.9706084\lambda^2 - 1.8875801\lambda + 0.3595220 = 0$$

$$F(-\lambda) = \lambda^3 + 4.9038526\lambda^2 - 9.5367138\lambda + 1.8164307 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 9.8077052\lambda - 9.5367138$$

$F(0.210\ \ 00534) = +0.0392417, \qquad F'(0.210) = 7.3448$

$F(0.21534\ \ 0000245) = 0.0001784 \qquad F'(0.21534) = 7.2856$

$F(0.2153645) = 0.K$

$\therefore \lambda = -0.2153645, \quad \lambda^2 = +0.0463819, \quad \lambda^3 = -0.0099890$

$$\frac{\sigma R}{Et} = 0.4952381 \times \frac{1.5692210}{2.5692210} + \frac{1}{1.7368272}\left\{0.0321311\lambda^3 + 0.1792917\lambda^2 + 0.0899908\lambda + 0.3603169\right\}$$

$$= 0.30248 + 0.20127 = \underline{0.50375}$$

$\gamma^2 = 0.456976$

(0.2679098)

$$\frac{\varepsilon R}{t} = \underline{0.5176}$$

$$\zeta = 0.2537641 + 0.5625\left\{0.001004098(-\lambda)^4 - 0.002008195(-\lambda)^3 + 0.078700483(-\lambda)^2 - 0.057891698(-\lambda) + 0.035796928\right\}$$

$$= \underline{0.2689299}$$

$\Theta = \underline{+0.0038076} \qquad \Phi = -0.126276$

$\boxed{\gamma = 0.676, \quad \xi = 2.000}$, $(\gamma\xi) = 1.352$

$(\gamma\xi)^2 = 1.827904$

$$1.4074861\lambda^3 + 1.2687709\lambda^2 - 3.6976905\lambda + 0.5107168 = 0$$

$$F(-\lambda) = \lambda^3 + 0.9014447\lambda^2 - 2.6271596\lambda + 0.3628574 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 1.8028894\lambda - 2.6271596$$

$F(0.144\ \underset{002700}{}) = +0.0062248$, $F'(0.144) = 2.3053$

$F(0.14670\ \underset{0000044}{}) = 0.0000101$ $F'(0.14670) = 2.298$

$\lambda = -0.1467044$, $\lambda^2 = +0.0215222$, $\lambda^3 = -0.0031574$

$$\frac{\sigma R}{Et} = 0.4952381 \times \frac{1.7065912}{2.7065912} + \frac{1}{1.8296557}\{0.2284880\lambda^3 + 1.2749630\lambda^2 + 0.3797126\lambda + 0.0914205\}$$

$= 0.31226 + 0.03412 = \underline{0.34638}$

(0.2043040)

$$\frac{\varepsilon R}{t} = \underline{0.04520} \qquad \gamma^2 = 0.456976$$

$$\mathcal{E} = 0.1199791 + 4\{0.007140250(-\lambda)^4 - 0.014280500(-\lambda)^3 + 0.112940213(-\lambda)^2 - 0.052987463(-\lambda) + 0.027419446\}.$$

$= \underline{0.2083187}$

$\Theta = +0.01965$ $\Phi = -0.052404$

$\boxed{\gamma = 0.676, \quad \xi = 3.500}$ $\qquad (\gamma\xi) = 2.366$

$(\gamma\xi)^2 = 5.597956$

$$4.3104261\lambda^3 - 0.5506392\lambda^2 - 5.5069555\lambda + 0.8735843 = 0$$

$$F(-\lambda) = \lambda^3 - 0.1277459\lambda^2 - 1.2775894\lambda + 0.2026677$$

$$F'(-\lambda) = 3\lambda^2 - 0.2554918\lambda - 1.2775894$$

$F(\underset{0.004254}{0.155}) = 0.0052961, \qquad F'(0.155) = 1.2451$

$F(\underset{0000047}{0.159254}) = +0.0000056, \qquad F'(0.159254) = 1.242$

$$\lambda = -0.1592587, \quad \lambda^2 = +0.0253633, \quad \lambda^3 = -0.0040393$$

$$\frac{\sigma R}{Et} = 0.4952381 \times \frac{1.6814826}{2.6814826} + \frac{1}{1.8126822}\left\{0.6997445\lambda^3 + 3.9045773\lambda^2 + 2.3497447\lambda + 0.3966941\right\}$$

$$= 0.31055 + 0.06547 = \underline{\underline{0.37602}}$$

(0.4828860)

$\gamma^2 = 0.456976$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.6949}}$$

$$\mathcal{E} = 0.1413910 + 12.25\left\{0.021867016(-\lambda)^4 - 0.043734031(-\lambda)^3 + 0.195115566(-\lambda)^2 - 0.080826175(-\lambda) + 0.027288177\right\}$$

$= \underline{0.3766157}$ $\qquad \Theta = -0.2200733 \qquad \Phi = -0.072988$

$$\varepsilon = +\left[\frac{\sigma}{E} + \frac{\nu\lambda}{E} + \frac{1}{2}m^2\left[\frac{1}{16}f_1^2 + \frac{1}{8}\left(\frac{1}{2}f_1 + f_2\right)^2\right]\right]$$

$$= (1-\nu^2)\frac{\sigma}{E} + (1+\nu)\frac{m^2}{2}\left[\frac{1}{16}f_1^2 + \frac{1}{32}f_1^2 + \frac{1}{8}f_1f_2 + \frac{1}{8}f_2^2\right]$$

$$= (1-\nu^2)\frac{\sigma}{E} + (1+\nu)\frac{m^2}{2}\left[\frac{3}{32}f_1^2 + \frac{1}{8}f_1f_2 + \frac{1}{8}f_2^2\right]$$

$$= (1-\nu^2)\frac{\sigma}{E} + (1+\nu)\frac{m^2}{16}f_1^2\left[\frac{3}{4} + \lambda + \lambda^2\right]$$

$$\frac{\varepsilon R}{t} = (1-\nu^2)\frac{\sigma R}{Et} + \frac{(1+\nu)}{16}\gamma\xi^2\left[\lambda^2 + \lambda + \frac{3}{4}\right]$$

$$\boxed{\frac{\varepsilon R}{t} = 0.91\frac{\sigma R}{Et} + 0.08125\,\xi(\gamma\xi)\left[\lambda^2 + \lambda + 0.75000\right]}$$

$$\boxed{\frac{\varepsilon R}{t} = \frac{\sigma R}{Et} + 0.0625\,\xi(\gamma\xi)\left[\lambda^2 + \lambda + 0.75000\right]}$$

$$\frac{w}{R} = f_0 + f_1\left(\cos\frac{mx}{R} + \frac{1}{9}\cos\frac{3mx}{R}\right)\left(\cos\frac{my}{R} - \frac{1}{4}\cos\frac{2my}{R}\right) + f_2\left(\cos\frac{2mx}{R} + \cos\frac{2my}{R}\right)$$

$$\frac{\partial w}{\partial x} = -m\left\{ f_1\left(\sin\frac{mx}{R} + \frac{1}{3}\sin\frac{3mx}{R}\right)\left(\cos\frac{my}{R} - \frac{1}{4}\cos\frac{2my}{R}\right) + 2f_2\sin\frac{2mx}{R}\right\}$$

$$\frac{\partial w}{\partial y} = -m\left\{ f_1\left(\cos\frac{mx}{R} + \frac{1}{9}\cos\frac{3mx}{R}\right)\left(\sin\frac{my}{R} - \frac{1}{2}\sin\frac{2my}{R}\right) + 2f_2\sin\frac{2my}{R}\right\}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2\left\{ f_1\left(\cos\frac{mx}{R} + \cos\frac{3mx}{R}\right)\left(\cos\frac{my}{R} - \frac{1}{4}\cos\frac{2my}{R}\right) + 4f_2\cos\frac{2mx}{R}\right\}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial y^2} = -\left(\frac{m}{R}\right)^2\left\{ f_1\left(\cos\frac{mx}{R} + \frac{1}{9}\cos\frac{3mx}{R}\right)\left(\cos\frac{my}{R} - \cos\frac{2my}{R}\right) + 4f_2\cos\frac{2my}{R}\right\}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x\partial y} = \left(\frac{m}{R}\right)^2\left\{ f_1\left(\sin\frac{mx}{R} + \frac{1}{3}\sin\frac{3mx}{R}\right)\left(\sin\frac{my}{R} - \frac{1}{2}\sin\frac{2my}{R}\right)\right\}$$

$$\Delta\Delta F = E\left(\frac{m}{R}\right)^2\Bigg[m^2 f_1^2\Bigg\{ \left(\sin\frac{mx}{R} + \frac{1}{3}\sin\frac{3mx}{R}\right)^2\left(\sin\frac{my}{R} - \frac{1}{2}\sin\frac{2my}{R}\right)^2 \qquad (I)$$

$$- \left(\cos\frac{mx}{R} + \cos\frac{3mx}{R}\right)\left(\cos\frac{mx}{R} + \frac{1}{9}\cos\frac{3mx}{R}\right)\left(\cos\frac{my}{R} - \frac{1}{4}\cos\frac{2my}{R}\right)\left(\cos\frac{my}{R} - \cos\frac{2my}{R}\right)\Bigg\}$$

$$- 2m^2 f_1 f_2\Bigg\{ \left(\cos\frac{mx}{R} + \cos\frac{3mx}{R}\right)\left(-\frac{1}{4} + \cos\frac{my}{R} + \cos\frac{3my}{R} - \frac{1}{4}\cos\frac{4my}{R}\right)$$

$$+ \left(\frac{10}{9}\cos\frac{mx}{R} + \cos\frac{3mx}{R} + \frac{1}{9}\cos\frac{5mx}{R}\right)\left(\cos\frac{my}{R} - \cos\frac{2my}{R}\right)\Bigg\} \qquad (II)$$

$$- 16 f_2^2 m^2 \cos\frac{2mx}{R}\cos\frac{2my}{R} + \left\{ f_1\left(\cos\frac{mx}{R} + \cos\frac{3mx}{R}\right)\left(\cos\frac{my}{R} - \frac{1}{4}\cos\frac{2my}{R}\right) + 4f_2\cos\frac{2mx}{R}\right\} \qquad (III)$$

$$I = \left(\sin^2\frac{m\pi x}{R} + \frac{2}{3}\sin\frac{m\pi x}{R}\sin\frac{3m\pi x}{R} + \frac{1}{9}\sin^2\frac{3m\pi x}{R}\right)\left(\sin^2\frac{m\pi y}{R} - \sin\frac{m\pi y}{R}\sin\frac{2m\pi y}{R} + \frac{1}{4}\sin^2\frac{2m\pi y}{R}\right)$$

$$-\left(\cos^2\frac{m\pi x}{R} + \frac{10}{9}\cos\frac{m\pi x}{R}\cos\frac{3m\pi x}{R} + \frac{1}{9}\cos^2\frac{3m\pi x}{R}\right)\left(\cos^2\frac{m\pi y}{R} - \frac{5}{4}\cos\frac{m\pi y}{R}\cos\frac{2m\pi y}{R} + \frac{1}{4}\cos^2\frac{2m\pi y}{R}\right)$$

$$= \frac{1}{4}\left(\frac{10}{9} - \frac{1}{3}\cos\frac{2m\pi x}{R} - \frac{2}{3}\cos\frac{4m\pi x}{R} - \frac{1}{9}\cos\frac{6m\pi x}{R}\right)\left(\frac{5}{4} - \cos\frac{m\pi y}{R} - \cos\frac{2m\pi y}{R} + \cos\frac{3m\pi y}{R} - \frac{1}{4}\cos\frac{4m\pi y}{R}\right)$$

$$-\frac{1}{4}\left(\frac{10}{9} + \frac{19}{9}\cos\frac{2m\pi x}{R} + \frac{10}{9}\cos\frac{4m\pi x}{R} + \frac{1}{9}\cos\frac{6m\pi x}{R}\right)\left(\frac{5}{4} - \frac{5}{4}\cos\frac{m\pi y}{R} + \cos\frac{2m\pi y}{R} - \frac{5}{4}\cos\frac{3m\pi y}{R} + \frac{1}{4}\cos\frac{4m\pi y}{R}\right)$$

$$= \frac{1}{4}\Bigg\{\frac{5}{4}\left(-\frac{22}{9}\cos\frac{2m\pi x}{R} - \frac{16}{9}\cos\frac{4m\pi x}{R} - \frac{2}{9}\cos\frac{6m\pi x}{R}\right)$$

$$+ \frac{1}{4}\cos\frac{m\pi y}{R}\left(\frac{10}{9} + \frac{107}{9}\cos\frac{2m\pi x}{R} + \frac{74}{9}\cos\frac{4m\pi x}{R} + \cos\frac{6m\pi x}{R}\right)$$

$$- \cos\frac{2m\pi y}{R}\left(\frac{20}{9} + \frac{16}{9}\cos\frac{2m\pi x}{R} + \frac{4}{9}\cos\frac{4m\pi x}{R}\right)$$

$$+ \frac{1}{4}\cos\frac{3m\pi y}{R}\left(10 + \frac{83}{9}\cos\frac{2m\pi x}{R} + \frac{26}{9}\cos\frac{4m\pi x}{R} + \frac{1}{9}\cos\frac{6m\pi x}{R}\right)$$

$$- \frac{1}{4}\cos\frac{4m\pi y}{R}\left(\frac{20}{9} + \frac{16}{9}\cos\frac{2m\pi x}{R} + \frac{4}{9}\cos\frac{4m\pi x}{R}\right)\Bigg\}$$

$$I = -\left\{ \frac{110}{144}\cos\frac{2m x}{R} + \frac{80}{144}\cos\frac{4m x}{R} + \frac{10}{144}\cos\frac{6m x}{R} - \frac{10}{144}\cos\frac{m y}{R} + \frac{20}{36}\cos\frac{2m y}{R} - \frac{10}{16}\cos\frac{3m y}{R} \right.$$

$$+ \frac{20}{144}\cos\frac{4m y}{R} - \frac{107}{144}\cos\frac{2m x}{R}\cos\frac{m y}{R} - \frac{74}{144}\cos\frac{4m x}{R}\cos\frac{m y}{R} - \frac{1}{16}\cos\frac{6m x}{R}\cos\frac{m y}{R}$$

$$+ \frac{16}{36}\cos\frac{2m x}{R}\cos\frac{2m y}{R} + \frac{4}{36}\cos\frac{4m x}{R}\cos\frac{2m y}{R} - \frac{83}{144}\cos\frac{2m x}{R}\cos\frac{3m y}{R} - \frac{26}{144}\cos\frac{4m y}{R}\cos\frac{3m y}{R}$$

$$\left. - \frac{1}{144}\cos\frac{6m x}{R}\cos\frac{3m y}{R} + \frac{16}{144}\cos\frac{2m x}{R}\cos\frac{4m y}{R} + \frac{4}{144}\cos\frac{4m x}{R}\cos\frac{4m y}{R} \right\}$$

$$II = -2\left\{ -\frac{1}{4}\cos\frac{m x}{R} - \frac{1}{4}\cos\frac{3m x}{R} + \cos\frac{m y}{R}\left(\frac{19}{9}\cos\frac{m x}{R} + 2\cos\frac{3m x}{R} + \frac{1}{9}\cos\frac{5m x}{R}\right) \right.$$

$$- \cos\frac{2m y}{R}\left(\frac{10}{9}\cos\frac{m x}{R} + \cos\frac{3m x}{R} + \frac{1}{9}\cos\frac{5m x}{R}\right) + \cos\frac{3m y}{R}\left(\cos\frac{m x}{R} + \cos\frac{3m x}{R}\right)$$

$$\left. - \frac{1}{4}\cos\frac{4m y}{R}\left(\cos\frac{m x}{R} + \cos\frac{3m x}{R}\right) \right\}$$

$$II = -\left\{ -\frac{1}{2}\cos\frac{m x}{R} - \frac{1}{2}\cos\frac{3m x}{R} + \frac{38}{9}\cos\frac{m x}{R}\cos\frac{m y}{R} + 4\cos\frac{3m x}{R}\cos\frac{m y}{R} + \frac{2}{9}\cos\frac{5m x}{R}\cos\frac{m y}{R} \right.$$

$$- \frac{20}{9}\cos\frac{m x}{R}\cos\frac{2m y}{R} - 2\cos\frac{3m x}{R}\cos\frac{2m y}{R} - \frac{2}{9}\cos\frac{5m x}{R}\cos\frac{2m y}{R} + 2\cos\frac{m x}{R}\cos\frac{3m y}{R} + 2\cos\frac{3m x}{R}\cos\frac{3m y}{R}$$

$$\left. - \frac{1}{2}\cos\frac{m x}{R}\cos\frac{4m y}{R} - \frac{1}{2}\cos\frac{3m x}{R}\cos\frac{4m y}{R} \right\}$$

$$\boxed{III = f_1\left(\cos\frac{m\pi x}{R}\cos\frac{m\pi y}{R} + \cos\frac{3m\pi x}{R}\cos\frac{m\pi y}{R} - \frac{1}{4}\cos\frac{m\pi x}{R}\cos\frac{2m\pi y}{R} - \frac{1}{4}\cos\frac{3m\pi x}{R}\cos\frac{2m\pi y}{R}\right) + 4f_2\cos\frac{2m\pi x}{R}}$$

$$\Delta\Delta F = -E\left(\frac{m\pi}{R}\right)^{-2}\Big[\frac{1}{16}\cos\frac{2m\pi x}{R}\left(\frac{110}{144}f_1^2m^2 - 4f_2\right) + \frac{1}{256}\cos\frac{4m\pi x}{R}\left(\frac{80}{144}f_1^2m^2\right) + \frac{1}{1296}\cos\frac{6m\pi x}{R}\left(\frac{10}{144}f_1^2m^2\right) - \frac{1}{1}\cos\frac{m\pi y}{R}\left(\frac{10}{144}f_1^2m^2\right)$$

$$+\frac{1}{16}\cos\frac{2m\pi y}{R}\left(\frac{20}{36}f_1^2m^2\right) - \frac{1}{81}\cos\frac{3m\pi y}{R}\left(\frac{10}{16}f_1^2m^2\right) + \frac{1}{256}\cos\frac{4m\pi y}{R}\left(\frac{20}{144}f_1^2m^2\right) - \frac{1}{25}\cos\frac{2m\pi x}{R}\cos\frac{m\pi y}{R}\left(\frac{107}{144}f_1^2m^2\right)$$

$$-\frac{1}{289}\cos\frac{4m\pi x}{R}\cos\frac{m\pi y}{R}\left(\frac{74}{144}f_1^2m^2\right) - \frac{1}{1369}\cos\frac{6m\pi x}{R}\cos\frac{m\pi y}{R}\left(\frac{1}{16}f_1^2m^2\right) + \frac{1}{64}\cos\frac{2m\pi x}{R}\cos\frac{2m\pi y}{R}\left(\frac{16}{36}f_1^2m^2 + 16f_2^2m^2\right)$$

$$+\frac{1}{400}\cos\frac{4m\pi x}{R}\cos\frac{2m\pi y}{R}\left(\frac{4}{36}f_1^2m^2\right) - \frac{1}{169}\cos\frac{2m\pi x}{R}\cos\frac{3m\pi y}{R}\left(\frac{83}{144}f_1^2m^2\right) - \frac{1}{625}\cos\frac{4m\pi x}{R}\cos\frac{3m\pi y}{R}\left(\frac{26}{144}f_1^2m^2\right)$$

$$-\frac{1}{2025}\cos\frac{6m\pi x}{R}\cos\frac{3m\pi y}{R}\left(\frac{1}{144}f_1^2m^2\right) + \frac{1}{400}\cos\frac{2m\pi x}{R}\cos\frac{4m\pi y}{R}\left(\frac{16}{144}f_1^2m^2\right) + \frac{1}{1024}\cos\frac{4m\pi x}{R}\cos\frac{4m\pi y}{R}\left(\frac{4}{144}f_1^2m^2\right)$$

$$-1\cos\frac{m\pi x}{R}\left(\frac{1}{2}f_1f_2m^2\right) - \frac{1}{81}\cos\frac{3m\pi x}{R}\left(\frac{1}{2}f_1f_2m^2\right) + \frac{1}{4}\cos\frac{m\pi x}{R}\cos\frac{m\pi y}{R}\left(\frac{38}{9}f_1f_2m^2 - f_1\right)$$

$$+\frac{1}{100}\cos\frac{3m\pi x}{R}\cos\frac{m\pi y}{R}\left(4f_1f_2m^2 - f_1\right) + \frac{1}{676}\cos\frac{5m\pi x}{R}\cos\frac{m\pi y}{R}\left(\frac{2}{9}f_1f_2m^2\right) - \frac{1}{25}\cos\frac{m\pi x}{R}\cos\frac{2m\pi y}{R}\left(\frac{20}{9}f_1f_2m^2 - \frac{1}{4}f_1\right)$$

$$-\frac{1}{169}\cos\frac{3m\pi x}{R}\cos\frac{2m\pi y}{R}\left(2f_1f_2m^2 - \frac{1}{4}f_1\right) - \frac{1}{841}\cos\frac{5m\pi x}{R}\cos\frac{2m\pi y}{R}\left(\frac{2}{9}f_1f_2m^2\right) + \frac{1}{100}\cos\frac{m\pi x}{R}\cos\frac{3m\pi y}{R}\left(2f_1f_2m^2\right)$$

$$+1\cos\frac{3m\pi x}{R}\cos\frac{3m\pi y}{R}\left(2f_1f_2m^2\right) - \frac{1}{289}\cos\frac{m\pi x}{R}\cos\frac{4m\pi y}{R}\left(\frac{1}{2}f_1f_2m^2\right) - \frac{1}{625}\cos\frac{3m\pi x}{R}\cos\frac{4m\pi y}{R}\left(\frac{1}{2}f_1f_2m^2\right)\Big]$$

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$$\rho_1 = \frac{2}{16}\left(\frac{110}{144}f_1^2 m^2 - 4f_1\right)^2 + \frac{2}{256}\left(\frac{80}{144}f_1^2 m^2\right)^2 + \frac{2}{1296}\left(\frac{10}{144}f_1^2 m^2\right)^2 + \frac{2}{16}\left(\frac{20}{36}f_1^2 m^2\right)^2 + 2\left(\frac{10}{144}f_1^2 m^2\right)^2$$

$$+ \frac{2}{81}\left(\frac{10}{16}f_1^2 m^2\right)^2 + \frac{2}{256}\left(\frac{20}{144}f_1^2 m^2\right)^2 + \frac{1}{25}\left(\frac{107}{144}f_1^2 m^2\right)^2 + \frac{1}{289}\left(\frac{74}{144}f_1^2 m^2\right)^2 + \frac{1}{1369}\left(\frac{1}{16}f_1^2 m^2\right)^2$$

$$+ \frac{1}{64}\left(\frac{16}{36}f_1^2 m^2 + 16 f_2^2 m^2\right)^2 + \frac{1}{400}\left(\frac{4}{36}f_1^2 m^2\right)^2 + \frac{1}{169}\left(\frac{83}{144}f_1^2 m^2\right)^2 + \frac{1}{625}\left(\frac{26}{144}f_1^2 m^2\right)^2$$

$$+ \frac{1}{2025}\left(\frac{1}{144}f_1^2 m^2\right)^2 + \frac{1}{400}\left(\frac{16}{144}f_1^2 m^2\right)^2 + \frac{1}{1024}\left(\frac{4}{144}f_1^2 m^2\right)^2 + 2\left(\frac{1}{2}f_1 f_2 m^2\right)^2 + \frac{2}{81}\left(\frac{1}{2}f_1 f_2 m^2\right)^2$$

$$+ \frac{1}{4}\left(\frac{38}{9}f_1 f_2 m^2 - f_1\right)^2 + \frac{1}{100}\left(-4 f_1 f_2 m^2 - f_1\right)^2 + \frac{1}{676}\left(\frac{2}{9}f_1 f_2 m^2\right)^2 + \frac{1}{25}\left(\frac{20}{9}f_1 f_2 m^2 - \frac{1}{4}f_1\right)^2$$

$$+ \frac{1}{169}\left(2 f_1 f_2 m^2 - \frac{1}{4}f_1\right)^2 + \frac{1}{841}\left(\frac{2}{9}f_1 f_2 m^2\right)^2 + \frac{1}{100}\left(2 f_1 f_2 m^2\right)^2 + \frac{1}{324}\left(2 f_1 f_2 m^2\right)^2 + \frac{1}{289}\left(\frac{1}{2}f_1 f_2 m^2\right)^2$$

$$+ \frac{1}{625}\left(\frac{1}{2}f_1 f_2 m^2\right)^2$$

$$\rho_1 = (f_1^2 m^2)^2 \frac{1}{20736}\left\{\frac{24200}{16} + \frac{12800}{256} + \frac{200}{1296} + 800 + 200 + 200 + \frac{800}{256} + \frac{11449}{25} + \frac{5476}{289} + \frac{81}{1369}\right.$$

$$\left. + 64 + \frac{16}{25} + \frac{6889}{169} + \frac{676}{625} + \frac{1}{2025} + \frac{256}{400} + \frac{16}{1024}\right\}$$

$$+ f_1^2 f_2^2 m^4 \left\{\frac{8}{36} + \frac{1}{2} + \frac{1}{162} + \frac{361}{81} + \frac{16}{100} + \frac{1}{13689} + \frac{16}{81} + \frac{4}{169} + \frac{4}{68121} + \frac{1}{25} + \frac{1}{81} + \frac{1}{1156}\right.$$

$$\left. + \frac{1}{2500}\right\} + 4(f_2^2 m^2)^2 - \left\{\frac{110}{144}f_1^2 f_2 m^2 + \frac{19}{9}f_1^2 f_2 m^2 + \frac{2}{25}f_1^2 f_2 m^2 + \frac{2}{45}f_1^2 f_2 m^2 + \frac{1}{169}f_1^2 f_2 m^2\right\}$$

$$+ \left\{2 f_2^2 + \frac{1}{4}f_1^2 + \frac{1}{100}f_1^2 + \frac{1}{400}f_1^2 + \frac{1}{2704}f_1^2\right\}$$

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$$\boxed{\wp_1 = 0.16154936\, f_1^4 m^4 + 5.6200987\, f_1^2 f_2^2 m^4 + 4\, f_2^4 m^4 - 3.0053616\, f_1^2 f_2 m^2 + 0.262869782\, f_1^2 + 2 f_2^2}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2 \left\{ f_1 \left(\cos\frac{mx}{R}\cos\frac{my}{R} + \cos\frac{3mx}{R}\cos\frac{my}{R} - \frac{1}{4}\cos\frac{mx}{R}\cos\frac{2my}{R} - \frac{1}{4}\cos\frac{3mx}{R}\cos\frac{2my}{R} \right) + 4 f_2 \cos\frac{2mx}{R} \right\}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial y^2} = -\left(\frac{m}{R}\right)^2 \left\{ f_1 \left(\cos\frac{mx}{R}\cos\frac{my}{R} + \frac{1}{9}\cos\frac{3mx}{R}\cos\frac{my}{R} - \cos\frac{mx}{R}\cos\frac{2my}{R} - \frac{1}{9}\cos\frac{3mx}{R}\cos\frac{2my}{R} \right) + 4 f_2 \cos\frac{2my}{R} \right\}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x \partial y} = \left(\frac{m}{R}\right)^2 \left\{ f_1 \left(\sin\frac{mx}{R}\sin\frac{my}{R} + \frac{1}{3}\sin\frac{3mx}{R}\sin\frac{my}{R} - \frac{1}{2}\sin\frac{mx}{R}\sin\frac{2my}{R} - \frac{1}{6}\sin\frac{3mx}{R}\sin\frac{2my}{R} \right\}$$

$$\wp_2 = \frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \left\{ 4 f_1^2 + \frac{100}{81} f_1^2 + \frac{25}{16} f_1^2 + \frac{169}{1296} f_1^2 + 64 f_2^2 \right\}$$

$$\boxed{\wp_2 = \frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \left\{ 6.9274691\, f_1^2 + 64 f_2^2 \right\}}$$

$$\frac{\partial w}{\partial y} = -m \left\{ f_1 \left(\cos\frac{mx}{R}\sin\frac{my}{R} + \frac{1}{9}\cos\frac{3mx}{R}\sin\frac{my}{R} - \frac{1}{2}\cos\frac{mx}{2}\sin\frac{2my}{R} - \frac{1}{18}\cos\frac{3mx}{R}\sin\frac{2my}{R} \right) + 2 f_2 \sin\frac{2my}{R} \right\}$$

$$\boxed{\frac{\lambda}{E} + \nu\frac{\sigma}{E} - \frac{1}{2}m^2 \left\{ f_1^2 \left(\frac{1}{4} + \frac{1}{324} + \frac{1}{16} + \frac{1}{1296} \right) + 2 f_2^2 \right\} + f_0 = 0}$$

$$\boxed{\frac{\lambda}{E} = m^2\left\{\frac{205}{1296}f_1^2 + f_2^2\right\} - f_0 - \nu\frac{\sigma}{E}}$$

$$\frac{\partial w}{\partial x} = -m\left\{f_1\left(\sin\frac{mx}{R}\cos\frac{my}{R} + \frac{1}{3}\sin\frac{3mx}{R}\cos\frac{my}{R} - \frac{1}{4}\sin\frac{mx}{R}\cos\frac{2my}{R} - \frac{1}{12}\sin\frac{3mx}{R}\cos\frac{2my}{R}\right) + 2f_2\sin\frac{2mx}{R}\right\}$$

$$g_0 = -8\frac{\sigma}{E}\left[\left(\frac{\sigma}{E}+\nu\frac{\lambda}{E}\right) + \frac{1}{2}m^2\left\{f_1^2\left(\frac{1}{4}+\frac{1}{36}+\frac{1}{64}+\frac{1}{576}\right) + 2f_2^2\right\}\right]$$

$$= -8\frac{\sigma}{E}\left[\left(\frac{\sigma}{E}+\nu\frac{\lambda}{E}\right) + m^2\left\{\frac{85}{576}f_1^2 + f_2^2\right\}\right]$$

$$= -8\frac{\sigma}{E}\left[(1-\nu^2)\left(\frac{\sigma}{E}\right) + m^2\left\{\left(\frac{85}{576}+\nu\frac{205}{1296}\right)f_1^2 + (1+\nu)f_2^2\right\} - \nu f_0\right]$$

$$g_0 = -4\left[2(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + m^2\left\{\left(\frac{85}{288}+\nu\frac{205}{648}\right)f_1^2 + 2(1+\nu)f_2^2\right\}\frac{\sigma}{E} - 2\nu f_0\frac{\sigma}{E}\right]$$

$$4\left[\left(\frac{\sigma}{E}\right)^2 + \left(\frac{\lambda}{E}\right)^2 + 2\nu\frac{\sigma}{E}\frac{\lambda}{E}\right] = 4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + m^4\left\{\frac{205}{1296}f_1^2+f_2^2\right\}^2 + f_0^2 - 2f_0m^2\left\{\frac{205}{1296}f_1^2+f_2^2\right\}\right.$$

$$\left. - 2\nu\frac{\sigma}{E}\left\{\frac{205}{1296}f_1^2+f_2^2\right\} + 2\nu f_0\frac{\sigma}{E} + 2\nu\frac{\sigma}{E}m^2\left\{\frac{205}{1296}f_1^2+f_2^2\right\} - 2\nu f_0\frac{\sigma}{E}\right]$$

$$K = -4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + m^2 \frac{\sigma}{E}\left\{ \left(\frac{85}{288} + \nu\frac{205}{648}\right) f_1^2 + 2(1+\nu) f_2^2 \right\} - 24\frac{\sigma}{E} f_0 \right.$$

$$\left. - m^4 \left\{ \frac{205}{1296} f_1^2 + f_2^2 \right\}^2 - f_0^2 + f_0 m^2 \left\{ \frac{205}{648} f_1^2 + 2 f_2^2 \right\} \right]$$

$$\therefore \quad \boxed{24\frac{\sigma}{E} + 2 f_0 - m^2 \left\{ \frac{205}{648} f_1^2 + 2 f_2^2 \right\} = 0}$$

$$K = -4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + m^2 \frac{\sigma}{E}\left\{ \left(\frac{85}{288} + \nu\frac{205}{648}\right) f_1^2 + 2(1+\nu) f_2^2 \right\} + f_0^2 - m^4 \left\{ \frac{205}{1296} f_1^2 + f_2^2 \right\}^2 \right]$$

$$\boxed{K = -4\left(\frac{\sigma}{E}\right)^2 - \frac{\sigma}{E} m^2 \left(\frac{85}{72} f_1^2 + 8 f_2^2 \right)}$$

$$\frac{85}{36}\frac{\sigma R}{Et}\gamma = (\gamma\xi)^2\left(11.2401974\rho^2 + 0.64619744\right) - (\gamma\xi)(6.0107232\rho) + 0.52573964 + \frac{1}{6(1-\nu^2)}\gamma^2\, 6.9274491$$

$$16\frac{\sigma R}{Et}\gamma\rho = (\gamma\xi)^2\left(16\rho^3 + 11.2401974\rho\right) - (\gamma\xi)(3.0053616) + 4\rho + \frac{1}{6(1-\nu^2)}\gamma^2\, 64\rho$$

$$\frac{\sigma R}{Et}\gamma = (\gamma\xi)^2\left(4.7605542\rho^2 + 0 + 0.27368362\right) - (\gamma\xi)(2.5457181\rho) + 0.22266620 + \frac{\gamma^2}{6(1-\nu^2)}\, 2.9339869$$

$$\frac{\sigma R}{Et}\gamma\rho = (\gamma\xi)^2\left(\rho^3 + 0.70251414\rho + 0\right) - (\gamma\xi)(0.18783558) + 0.25\rho + \frac{\gamma^2}{6(1-\nu^2)}\, 4\rho$$

$$(\gamma\xi)^2\left(3.7605542\rho^3 - 0.42883052\rho\right) - (\gamma\xi)\left(2.5457181\rho^2 - 0.18783558\right) - 0.02733380\rho - \frac{\gamma^2}{6(1-\nu^2)}\, 1.0660131\rho = 0$$

Thus

$$\left\{3.7605542(\gamma\xi)^2\right\}\rho^3 - \left\{2.5457181(\gamma\xi)\right\}\rho^2 - \left\{0.42883052(\gamma\xi)^2 + 0.02733380 + 0.19524049\gamma^2\right\}\rho + 0.18783558(\gamma\xi) = 0$$

$$(\gamma\xi)^2\rho^3 - 0.6769529(\gamma\xi)\rho^2 - \left\{0.1140339(\gamma\xi)^2 + 0.05191801\gamma^2 + 0.007268556\right\}\rho + 0.04994891(\gamma\xi) = 0$$

$$\frac{\sigma R}{Et} = \frac{1}{\gamma}\left\{(\gamma\xi)^2\left(4.7605542\rho^2 + 0.27368362\right) - 2.5457181(\gamma\xi)\rho + 0.22266620\right\} + 0.53736021$$

$$\left(\frac{w}{R}\right)_{\substack{at\ x=0\\ y=0}} = f_0 + f_1\left(\frac{10}{9}\times\frac{3}{4}\right) + f_2(2)$$

$$\left(\frac{w}{R}\right)_{\substack{at\ x=0\\ y=\frac{R\pi}{n}}} = f_0 - f_1\left(\frac{10}{9}\times\frac{5}{4}\right) + f_2(0)$$

$$\text{Amplitude} = \frac{30+50}{36}f_1 = \frac{80}{36}f_1 = \boxed{\frac{20}{9}f_1}$$

$\gamma = 0.100 \quad \xi = (16) \times \frac{9}{20} = 7.2$

$(\gamma\xi) = 0.72, \quad (\gamma\xi)^2 = 0.5184$

$0.5184\,\rho^3 - 0.4874061\,\rho^2 - 0.0669019\,\rho + 0.0359632 = 0$

$F(\rho) = \rho^3 - 0.9402124\,\rho^2 - 0.1290565\,\rho + 0.0693735 = 0$

$F'(\rho) = 3\rho^3 - 1.8804248\,\rho - 0.1290565$

$F(0.235) = +0.0000999, \quad F'(0.235) = 0.40528$

000246

$F(0.235246) =$ o.K. $\quad \rho = +0.235246, \quad \rho^2 = 0.0553407$

$$\frac{\sigma R}{Et} = 10\left\{0.5375645(\gamma\xi)^2 - 0.5988700(\gamma\xi) + 0.22266620\right\} + 0.05373602$$

$= \underline{\underline{0.75527}}$

$\boxed{\gamma = 0.081 \qquad \xi = 7.2}$ $\qquad \gamma^2 = 0.001561$

$(\gamma\xi) = 0.5832, \quad (\gamma\xi)^2 = 0.34012224$

$0.34012224\, \rho^3 - 0.3947989\, \rho^2 - 0.04639466\, \rho + 0.02913020 = 0$

$F(\rho) = \rho^3 - 1.1607559\, \rho^2 - 0.1364058\, \rho + 0.0856463 = 0$

$F'(\rho) = 3\rho^2 - 2.3215118\, \rho - 0.1364058$

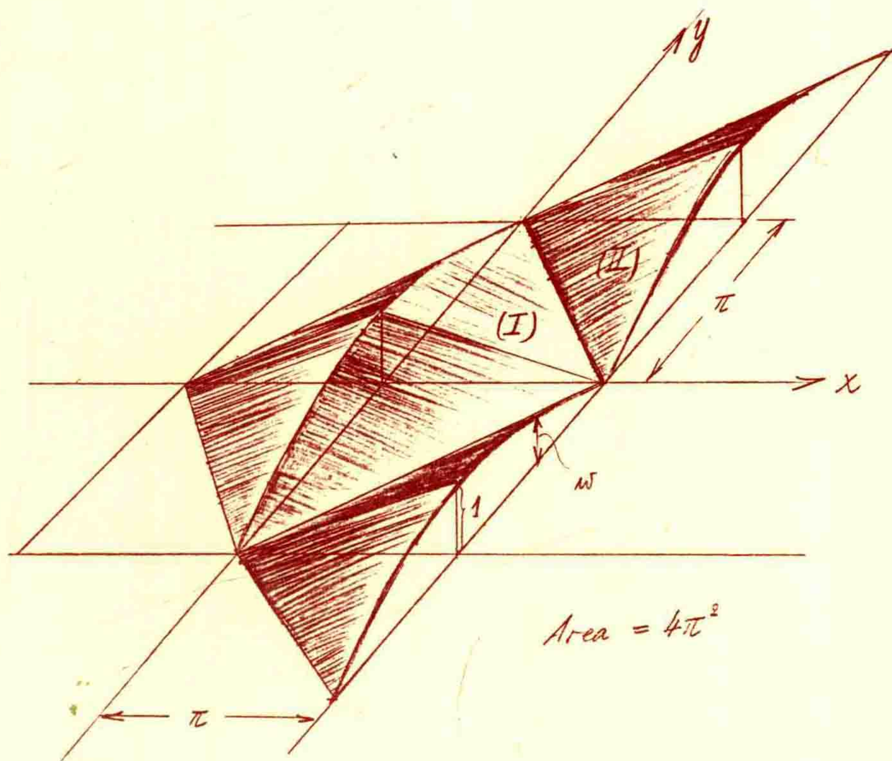
$F(0.239) = 0.0003937 \qquad F'(0.239) = 0.51988$

0007573

$F(0.2397573) = $ O.K. $\qquad \rho = +0.2392573, \quad \rho^2 = 0.05748356$

$$\frac{\sigma R}{Et} = \frac{1}{0.081}\left\{0.5473372(\gamma\xi)^2 - 0.6103545(\gamma\xi) + 0.2226620\right\} + 0.0435262$$

$= \underline{\underline{0.69623}}$



Area $= 4\pi^2$

$$w = \left\{1 - \left(\frac{y}{\pi}\right)^2\right\}\left\{1 - \frac{(x/\pi)}{1 - \left(\frac{y}{\pi}\right)}\right\}$$

$$= \left\{1 - \left(\frac{y}{\pi}\right)^2\right\} - \left(1 + \frac{y}{\pi}\right)\left(\frac{x}{\pi}\right) \quad \text{for (I)}$$

$$w = \left\{1 - \left(1 - \frac{y}{\pi}\right)^2\right\} \frac{\left(\frac{x}{\pi}\right) - \left(1 - \frac{y}{\pi}\right)}{\left(\frac{y}{\pi}\right)}$$

$$= \left(2 - \frac{y}{\pi}\right)\left(\frac{x}{\pi} + \frac{y}{\pi} - 1\right) \quad \text{for (II)}$$

$$w = a_{00} + a_{10}\cos x + a_{20}\cos 2x + a_{30}\cos 3x + a_{01}\cos y + a_{02}\cos 2y$$
$$+ a_{03}\cos 3y + a_{11}\cos x\cos y + a_{12}\cos x\cos 2y + a_{13}\cos x\cos 3y + \cdots$$

$$a_{00} = \frac{8}{4\pi^2}\int_0^\pi\int_0^{(\pi-x)}\left[\left\{1-\left(\frac{y}{\pi}\right)^2\right\}-\left(\frac{x}{\pi}\right)\left(1+\frac{y}{\pi}\right)\right]dy\,dx$$

$$= \frac{2}{\pi^2}\int_0^\pi\left[\left\{y-\frac{1}{3}y\left(\frac{y}{\pi}\right)^2\right\}-\left(\frac{x}{\pi}\right)\left(y+\frac{1}{2}y\frac{y}{\pi}\right)\right]_0^{\pi-x}dx$$

$$= \frac{2}{\pi}\int_0^\pi\left[\left(1-\frac{x}{\pi}\right)\left\{1-\frac{1}{3}\left(1-\frac{x}{\pi}\right)^2\right\}-\left(\frac{x}{\pi}\right)\left(1-\frac{x}{\pi}\right)\left\{1+\frac{1}{2}\left(1-\frac{x}{\pi}\right)\right\}\right]dx$$

$$= 2\int_0^1\left[\eta\left\{1-\frac{1}{3}\eta^2\right\}+\eta^2\left(1+\frac{1}{2}\eta\right)-\eta\left(1+\frac{1}{2}\eta\right)\right]d\eta$$

$$= 2\int_0^1\left(\eta-\frac{1}{3}\eta^3+\eta^2+\frac{1}{2}\eta^3-\eta-\frac{1}{2}\eta^2\right)d\eta$$

$$= 2\int_0^1\left(\frac{1}{2}\eta^2+\frac{1}{6}\eta^3\right)d\eta = 2\left[\frac{1}{6}+\frac{1}{24}\right] = \frac{5}{12} = \underline{\underline{\frac{5}{12}}}$$

$$a_{10} = \frac{2}{\pi^2}\left[\int_0^\pi\int_0^{(\pi-x)}\left[\left\{1-\left(\frac{y}{\pi}\right)^2\right\}-\left(\frac{x}{\pi}\right)\left(1+\frac{y}{\pi}\right)\right]\cos x\,dy\,dx\right.$$

$$\left.+\int_0^\pi\int_{\pi-x}^\pi\left(2-\frac{y}{\pi}\right)\left(\frac{x}{\pi}+\frac{y}{\pi}-1\right)\cos x\,dy\,dx\right]$$

$$= \frac{2}{\pi}\left\{\int_0^\pi\left[\frac{1}{2}\left(1-\frac{x}{\pi}\right)^2+\frac{1}{6}\left(1-\frac{x}{\pi}\right)^3\right]\cos x\,dx\right.$$

$$\left.+\int_0^\pi\left[\frac{2}{3}-\frac{3}{2}\left(1-\frac{x}{\pi}\right)+\left(1-\frac{x}{\pi}\right)^2-\frac{1}{6}\left(1-\frac{x}{\pi}\right)^3\right]\cos x\,dx\right\}$$

$$= \frac{2}{\pi}\left\{\int_0^\pi\left\{\frac{2}{3}-\frac{3}{2}\left(1-\frac{x}{\pi}\right)+\frac{3}{2}\left(1-\frac{x}{\pi}\right)^2\right\}\cos x\,dx\right.$$

now $\int_0^\pi (1-\frac{x}{\pi})^n \cos x\, dx = \pi \int_0^1 (1-\xi)^n \cos \pi\xi\, d\xi$

$= -\pi \int_0^1 \eta^n \cos \eta\pi\, d\eta = -\frac{1}{\pi^n} \int_0^\pi \zeta^n \cos\zeta\, d\zeta$

$a_{10} = -2\left\{ \frac{1}{2\pi^3}\cdot(-2\pi) + \frac{1}{6\pi^4}\left(-(3\pi^2-6)+6\right) - \frac{3}{2\pi^2}(-1-1) \right.$

$\left. + \frac{1}{\pi^3}(-2\pi) - \frac{1}{6\pi^4}\left(-(3\pi^2-6)+6\right) \right\}$

$= -2\left\{ -\frac{3}{\pi^2} + \frac{3}{\pi^2} \right\} = 0 = a_{10}$

$\int_0^\pi (1-\frac{x}{\pi})^n \cos 2x\, dx = + \frac{\pi}{(2\pi)^{n+1}} \int_0^{2\pi} \zeta^n \cos\zeta\, d\zeta$

$a_{20} = +2\left\{ -\frac{3}{2}\frac{1}{4\pi^2}(0) + \frac{3}{2}\frac{1}{8\pi^3}(4\pi) \right\} = +\frac{3}{2\pi^2} = a_{20}$

$a_{30} = -2\left\{ -\frac{3}{2}\frac{1}{9\pi^2}(-1-1) + \frac{3}{2}\frac{1}{27\pi^3}(-6\pi) \right\}$

$= -2\left\{ \frac{1}{3\pi^2} - \frac{1}{3\pi^2} \right\} = 0 = a_{30}$

$$a_{01} = \frac{2}{\pi^2}\Big[\int_0^{\pi}\int_0^{\pi-y}\Big[\Big\{1-\big(\tfrac{y}{\pi}\big)^2\Big\}-\big(\tfrac{x}{\pi}\big)\big(1+\tfrac{y}{\pi}\big)\Big]dx\cos y\,dy$$

$$+\int_0^{\pi}\int_{\pi-y}^{\pi}\Big[2\big(\tfrac{x}{\pi}-1+\tfrac{y}{\pi}\big)-\tfrac{y}{\pi}\big(\tfrac{x}{\pi}-1+\tfrac{y}{\pi}\big)\Big]dx\cdot\cos y\,dy$$

$$=\frac{2}{\pi}\Big[\int_0^{\pi}\Big[\big(1-\tfrac{y}{\pi}\big)\big(1-\big(\tfrac{y}{\pi}\big)^2\big)-\tfrac{1}{2}\big(1+\tfrac{y}{\pi}\big)\big(1-\tfrac{y}{\pi}\big)^2\Big]\cos y\,dy$$

$$+\int_0^{\pi}\Big[2\big(\tfrac{1}{2}-1+\tfrac{y}{\pi}\big)-\tfrac{y}{\pi}\big(\tfrac{1}{2}-1+\tfrac{y}{\pi}\big)-2\big(1-\tfrac{y}{\pi}\big)\big(\tfrac{1}{2}\big(1-\tfrac{y}{\pi}\big)-1+\tfrac{y}{\pi}\big)$$

$$+\big(1-\tfrac{y}{\pi}\big)\tfrac{y}{\pi}\big(\tfrac{1}{2}\big(1-\tfrac{y}{\pi}\big)-1+\tfrac{y}{\pi}\big)\Big]\cos y\,dy$$

$$=\frac{2}{\pi}\int_0^{\pi}\cos y\Big[\tfrac{1}{2}\big(1-\tfrac{y}{\pi}\big)\big(1-\big(\tfrac{y}{\pi}\big)^2\big)+\big(2-\tfrac{y}{\pi}\big)\big(\tfrac{y}{\pi}-\tfrac{1}{2}\big)$$

$$+\tfrac{1}{2}\big(2-\tfrac{y}{\pi}\big)\big(1-\tfrac{y}{\pi}\big)\big(1-\tfrac{y}{\pi}\big)\Big]dy$$

$$=\frac{2}{\pi}\int_0^{\pi}\cos y\Big[\tfrac{1}{2}-\tfrac{1}{2}\tfrac{y}{\pi}-\tfrac{1}{2}\big(\tfrac{y}{\pi}\big)^2+\tfrac{1}{2}\big(\tfrac{y}{\pi}\big)^3+2\big(\tfrac{y}{\pi}\big)-\big(\tfrac{y}{\pi}\big)^2-1+\tfrac{1}{2}\big(\tfrac{y}{\pi}\big)$$

$$1-2\tfrac{y}{\pi}+\big(\tfrac{y}{\pi}\big)^2-\tfrac{1}{2}\big(\tfrac{y}{\pi}\big)+\big(\tfrac{y}{\pi}\big)^2-\tfrac{1}{2}\big(\tfrac{y}{\pi}\big)^3\Big]dy$$

$$=\frac{2}{\pi}\int_0^{\pi}\Big[\tfrac{1}{2}-\tfrac{1}{2}\big(\tfrac{y}{\pi}\big)+\tfrac{1}{2}\big(\tfrac{y}{\pi}\big)^2\Big]\cos y\,dy$$

$$=\frac{1}{\pi}\int_0^{\pi}\Big[1-\big(\tfrac{y}{\pi}\big)+\big(\tfrac{y}{\pi}\big)^2\Big]\cos y\,dy$$

$$a_{01} = \frac{1}{\pi}\left[-\frac{1}{\pi}(-1-1) + \frac{1}{\pi^2}(-2\pi)\right] = 0,$$

6f2

$$a_{02} = \frac{1}{\pi}\left[-\frac{1}{4\pi}(1-1) + \frac{1}{8\pi^2}(4\pi)\right] = \frac{1}{2\pi^2} = a_{02}$$

$$a_{03} = \frac{1}{\pi}\left[-\frac{1}{9\pi}(-1-1) + \frac{1}{27\pi^2}(-6\pi)\right] = 0.$$

$$a_{04} = \frac{1}{\pi}\left[-\frac{1}{16\pi}(0) + \frac{1}{64\pi^2}(8\pi)\right] = +\frac{1}{8\pi^2} = a_{04}$$

$$a_{05} = \frac{1}{\pi}\left[-\frac{1}{25\pi}(-1-1) + \frac{1}{125\pi^2}(-10\pi)\right] = 0$$

$$a_{06} = \frac{1}{\pi}\left[0 + \frac{1}{36\times 6\pi^2}(12\pi)\right] = +\frac{1}{18\pi^2} = a_{06}$$

$$a_{40} = \frac{3}{8\pi^2}$$

$$a_{60} = \frac{3}{18\pi^2}$$

$$2 - \frac{v}{\pi} + 1$$

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$$a_{11} = \frac{4}{\pi^2}\left[\int_0^\pi\int_0^{\pi-x}\left[\left\{1-\left(\frac{y}{\pi}\right)^2\right\}-\left(\frac{x}{\pi}\right)\left\{1+\left(\frac{y}{\pi}\right)\right\}\right]\cos y\,dy\,\cos x\,dx\right.$$

$$\left.+\int_0^\pi\int_{\pi-x}^\pi\left(2-\frac{y}{\pi}\right)\left(\frac{x}{\pi}-1+\frac{y}{\pi}\right)\cos y\,dy\,\cos x\,dx\right]$$

$$=\frac{4}{\pi^2}\left[\int_0^\pi\int_0^{\pi-x}\left\{\left(1-\frac{x}{\pi}\right)-\left(\frac{x}{\pi}\right)\left(\frac{y}{\pi}\right)-\left(\frac{y}{\pi}\right)^2\right\}\cos y\,dy\,\cos x\,dx\right.$$

$$\left.+\int_0^\pi\int_{\pi-x}^\pi\left\{2\left(\frac{x}{\pi}-1\right)+\left(3-\frac{x}{\pi}\right)\frac{y}{\pi}-\left(\frac{y}{\pi}\right)^2\right\}\cos y\,dy\,\cos x\,dx\right]$$

$$=\frac{4}{\pi^2}\left[\int_0^\pi\left\{\left(1-\frac{x}{\pi}\right)\sin x-\frac{x}{\pi}\frac{1}{\pi}\left(-\cos x-1+\sin x(\pi-x)\right)\right.\right.$$

$$\left.-\frac{1}{\pi^2}\left(-\cancel{2(\pi-x)\cos x}+\cancel{\left((\pi-x)^2-2\right)\sin x}\right)\right\}\cos x\,dx$$

$$+\int_0^\pi\left\{-\cancel{2\left(\frac{x}{\pi}-1\right)\sin x}+\left(3-\frac{x}{\pi}\right)\frac{1}{\pi}\left(-1+\cos x-(\pi-x)\sin x\right)\right.$$

$$\left.\left.-\frac{1}{\pi^2}\left(-2\pi+\cancel{2(\pi-x)\cos x}-\cancel{\left((\pi-x)^2-2\right)\sin x}\right)\right\}\cos x\,dx\right]$$

$$=\frac{4}{\pi^2}\int_0^\pi\left\{3\left(1-\frac{x}{\pi}\right)\sin x-\frac{x}{\pi}\frac{1}{\pi}(-2)+\frac{3}{\pi}\left(-1+\cos x-(\pi-x)\sin x\right)\right\}\cos x\,dx$$

$$=\frac{4}{\pi^2}\int_0^\pi\left\{\cancel{3\left(1-\frac{x}{\pi}\right)\sin x}+\frac{2}{\pi}\frac{x}{\pi}-\frac{3}{\pi}+\frac{3}{\pi}\cos x-\cancel{3\left(1-\frac{x}{\pi}\right)\sin x}\right\}\cos x\,dx$$

$$=\frac{4}{\pi^2}\int_0^\pi\frac{1}{\pi}\left\{2\frac{x}{\pi}-3+3\cos x\right\}\cos x\,dx$$

684

$$a_{11} = \frac{4}{\pi^2}\left\{\frac{3}{2} + \frac{2}{\pi^2}\int_0^{\pi} x\cos x\,dx\right\} = \frac{4}{\pi^2}\left\{\frac{3}{2} - \frac{4}{\pi^2}\right\}$$

$$a_{21} = \frac{4}{\pi^2}\int_0^{\pi}\frac{1}{\pi}\left\{2\frac{x}{\pi} - 3 + 3\cos x\right\}\cos 2x\,dx$$

$$= \frac{8}{\pi^4}\int_0^{\pi} x\cos 2x\,dx = \frac{2}{\pi^4}\int_0^{2\pi}\theta\cos\theta\,d\theta$$

$$= \frac{2}{\pi^4}\left\{\ 0\ \right\} = 0$$

$$a_{31} = \frac{4}{\pi^2}\int_0^{\pi}\frac{1}{\pi}\left\{2\frac{x}{\pi} - 3 + 3\cos x\right\}\cos 3x\,dx$$

$$= \frac{8}{\pi^4}\int_0^{\pi} x\cos 3x\,dx = \frac{8}{9\pi^4}\int_0^{3\pi}\theta\cos\theta\,d\theta$$

$$= -\frac{16}{9\pi^4}$$

$$\int_0^{\pi-x}\left\{\left(1-\frac{x}{\pi}\right)-\left(\frac{x}{\pi}\right)\frac{y}{\pi}-\left(\frac{y}{\pi}\right)^2\right\}\cos 2y\,dy$$

$$+\int_{\pi-x}^{\pi}\left\{2\left(\frac{x}{\pi}-1\right)+\left(3-\frac{x}{\pi}\right)\frac{y}{\pi}-\left(\frac{y}{\pi}\right)^2\right\}\cos 2y\,dy$$

$$=-\frac{1}{2}^{\left(1-\frac{x}{\pi}\right)}\sin 2x-\left(\frac{x}{\pi}\right)\frac{1}{4\pi}\left\{(\cos 2x-1)+(-2(\pi-x)\sin 2x)\right\}$$

$$-\frac{1}{8\pi^2}\left\{4(\pi-x)\cos 2x-\left(4(\pi-x)^2-2\right)\sin 2x\right\}$$

$$+\left(\frac{x}{\pi}-1\right)\left\{+\sin 2x\right\}+\left(3-\frac{x}{\pi}\right)\frac{1}{4\pi}\left\{1-\cos 2x+2(\pi-x)\sin 2x\right\}$$

$$-\frac{1}{8\pi^2}\left\{4\pi-4(\pi-x)\cos 2x+\left\{4(\pi-x)^2-2\right\}\sin 2x\right\}$$

$$=-\frac{3}{2}\left(1-\frac{x}{\pi}\right)\sin 2x+\frac{3}{2}\left(1-\frac{x}{\pi}\right)\sin 2x-\frac{1}{2\pi}+\frac{3}{4\pi}(1-\cos 2x)$$

$$a_{12}=\frac{4}{\pi^2}\int_0^{\pi}-\frac{3}{4\pi}\cos 2x\cos x\,dx=0$$

$$a_{22}=\frac{4}{\pi^2}\int_0^{\pi}-\frac{3}{4\pi}\cos^2 2x\,dx=-\frac{4}{\pi^2}\frac{3}{8}=-\frac{3}{2\pi^2}$$

$$a_{32}=0.$$

$$\int_0^{\pi-x}\left\{\left(1-\frac{x}{\pi}\right)-\left(\frac{x}{\pi}\right)\frac{y}{\pi}-\left(\frac{y}{\pi}\right)^2\right\}\cos 3y\,dy$$

$$+\int_{\pi-x}^{\pi}\left\{2\left(\frac{x}{\pi}-1\right)+\left(3-\frac{x}{\pi}\right)\frac{y}{\pi}-\left(\frac{y}{\pi}\right)^2\right\}\cos 3y\,dy$$

$$=\frac{1}{3}\left(1-\frac{x}{\pi}\right)\left\{\sin 3x\right\}-\left(\frac{x}{\pi}\right)\frac{1}{9\pi}\left\{-\cos 3x-1+3(\pi-x)\sin 3x\right\}$$

$$-\frac{1}{27\pi^2}\left\{-6(\pi-x)\cos 3x+\left\{9(\pi-x)^2-2\right\}\sin 3x\right\}$$

$$+\frac{2}{3}\left(\frac{x}{\pi}-1\right)\left\{-\sin 3x\right\}+\left(3-\frac{x}{\pi}\right)\frac{1}{9\pi}\left\{-1+\cos 3x-3(\pi-x)\sin 3x\right\}$$

$$-\frac{1}{27\pi^2}\left\{-6\pi+6(\pi-x)\cos 3x-\left\{9(\pi-x)^2-2\right\}\sin 3x\right\}$$

$$=\left(1-\frac{x}{\pi}\right)\sin 3x+\frac{2}{9\pi}\frac{x}{\pi}-\frac{1}{3\pi}+\frac{1}{3\pi}\cos 3x-\left(1-\frac{x}{\pi}\right)\sin 3x$$

$$=-\frac{1}{3\pi}+\frac{1}{3\pi}\cos 3x+\frac{2}{9\pi}\frac{x}{\pi}$$

$$a_{13}=\frac{4}{\pi^2}\frac{2}{9\pi}\int_0^{\pi}\frac{x}{\pi}\cos x\,dx=\frac{8}{9\pi^4}\int_0^{\pi}x\cos x\,dx$$

$$=-\frac{16}{9\pi^4}$$

$$a_{23}=\frac{4}{\pi^2}\frac{2}{9\pi}\int_0^{\pi}\frac{x}{\pi}\cos 2x\,dx=0$$

$$a_{33}=\frac{4}{\pi^2}\int_0^{\pi}\left(\frac{1}{3\pi}\cos 3x+\frac{2}{9\pi}\frac{x}{\pi}\right)\cos 3x\,dx=\frac{4}{\pi^2}\left\{\frac{1}{6}-\frac{2}{9\pi^2}\frac{2}{9}\right\}$$

$a_{00} = \frac{5}{12}$

$a_{10} = 0$

$a_{20} = \frac{1}{\pi^2}\left(\frac{3}{2}\right)$

$a_{30} = 0$ $\qquad a_{40} = \frac{1}{\pi^2}\left(\frac{3}{8}\right)$ $\qquad a_{60} = \frac{1}{\pi^2}\left(\frac{3}{18}\right)$

$a_{01} = 0$

$a_{02} = \frac{1}{\pi^2}\left(\frac{1}{2}\right)$

$a_{03} = 0$ $\qquad a_{40} = \frac{1}{\pi^2}\left(\frac{1}{8}\right)$ $\qquad a_{60} = \frac{1}{\pi^2}\left(\frac{1}{18}\right)$

$a_{11} = \frac{1}{\pi^2}\left(6 - \frac{16}{\pi^2}\right)$, $\qquad a_{21} = 0$, $\qquad a_{31} = \frac{1}{\pi^2}\left(-\frac{16}{9\pi^2}\right)$

$a_{12} = 0$, $\qquad a_{22} = \frac{1}{\pi^2}\left(-\frac{3}{2}\right)$, $\qquad a_{32} = 0$

$a_{13} = \frac{1}{\pi^2}\left(-\frac{16}{9\pi^2}\right)$, $\qquad a_{23} = 0$, $\qquad a_{33} = \frac{1}{\pi^2}\left(\frac{2}{3} - \frac{16}{81\pi^2}\right)$

$$
\begin{aligned}
\frac{w}{R} = {} & \frac{1}{4}\cos\frac{2\pi x}{R} + \frac{1}{16}\cos\frac{4\pi x}{R} + \frac{1}{36}\cos\frac{6\pi x}{R} \\
& + \frac{1}{12}\cos\frac{2\pi y}{R} + \frac{1}{48}\cos\frac{4\pi y}{R} + \frac{1}{108}\cos\frac{6\pi y}{R} \\
& + \left(1 - \frac{8}{3\pi^2}\right)\cos\frac{\pi x}{R}\cos\frac{\pi y}{R} - \frac{8}{27\pi^2}\cos\frac{3\pi x}{R}\cos\frac{\pi y}{R} - \frac{1}{4}\cos\frac{2\pi x}{R}\cos\frac{2\pi y}{R} \\
& - \frac{8}{27\pi^2}\cos\frac{\pi x}{R}\cos\frac{3\pi y}{R} + \left(\frac{1}{9} - \frac{8}{243\pi^2}\right)\cos\frac{3\pi x}{R}\cos\frac{3\pi y}{R}
\end{aligned}
$$

$$\frac{w}{R} = 0.250000 \cos\frac{2\pi x}{R} + 0.0625000 \cos\frac{4\pi x}{R} + 0.02777778 \cos\frac{6\pi x}{R}$$

$$+ 0.08333333 \cos\frac{2\pi y}{R} + 0.02083333 \cos\frac{4\pi y}{R} + 0.009259259 \cos\frac{6\pi y}{R}$$

$$+ 0.7298102 \cos\frac{\pi x}{R} \cos\frac{\pi y}{R} - 0.03002109 \cos\frac{3\pi x}{R} \cos\frac{\pi y}{R} - 0.25 \cos\frac{2\pi x}{R} \cos\frac{2\pi y}{R}$$

$$- 0.03002109 \cos\frac{\pi x}{R} \cos\frac{3\pi y}{R} + 0.10777543 \cos\frac{3\pi x}{R} \cos\frac{3\pi y}{R}$$

$$\frac{w}{R} = \left\{ 0.2500000 \cos\frac{2\pi x}{R} + 0.06250000 \cos\frac{4\pi x}{R} + 0.02777778 \cos\frac{6\pi x}{R} \right\}$$

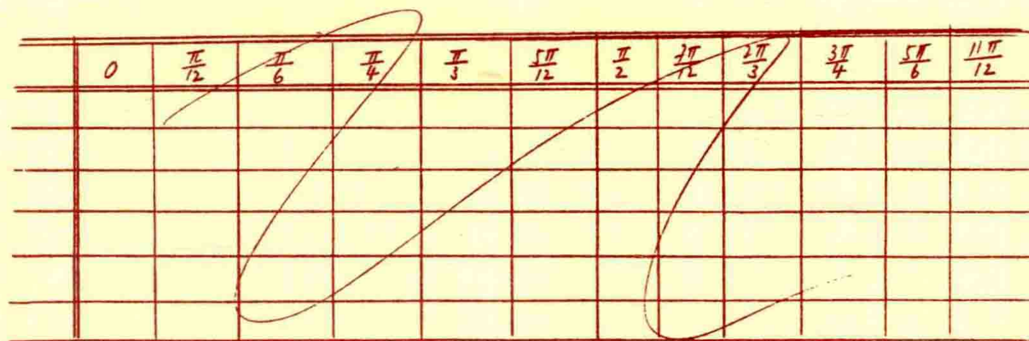
$$+ \left\{ 0.7298102 \cos\frac{\pi x}{R} - 0.03002109 \cos\frac{3\pi x}{R} \right\} \cos\frac{\pi y}{R}$$

$$+ \left\{ 0.08333333 - 0.25 \cos\frac{2\pi x}{R} \right\} \cos\frac{2\pi y}{R}$$

$$+ \left\{ -0.03002109 \cos\frac{\pi x}{R} + 0.10777543 \cos\frac{3\pi x}{R} \right\} \cos\frac{3\pi y}{R}$$

$$+ 0.02083333 \cos\frac{4\pi y}{R}$$

$$+ 0.009259259 \cos\frac{6\pi y}{R}$$



	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$

	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\cos\theta$	1	0.9659	0.8660	0.7071	0.5000	0.2588	0	−0.2588	−0.5000	−0.7071	−0.8660	−0.9659	−1.0000
$\cos 2\theta$	1	0.8660	0.50000	0	−0.5000	−0.8660	−1.0000	−0.8660	−0.5000	0	+0.50000	+0.8660	1
$\cos 3\theta$	1	0.7071	0	−0.7071	−1.000	−0.7071	0	+0.7071	1	+0.7071	0	−0.7071	−1.0000
$\cos 4\theta$	1	0.5000	−0.5000	−1.0000	−0.5000	+0.5000	+1	+0.5000	−0.5000	−1.000	−0.5000	+0.5000	1
$\cos 6\theta$	1	0	−1	0	1	0	−1	0	1	0	−1	0	1

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$x = 0$

$$\frac{w}{R} = 0.34027778 + 0.69978911 \cos\frac{\pi y}{R} - 0.166666667 \cos\frac{2\pi y}{R} + 0.07775434 \cos\frac{3\pi y}{R} + 0.02083333 \cos\frac{4\pi y}{R} + 0.009259259 \cos\frac{6\pi y}{R}$$

	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$a_1 \cos\theta$	0.69979	0.67593	0.60602	0.49482	0.34990	0.18111	0	-0.18111	-0.34990	-0.49482	-0.60602	-0.67593	-0.69979
$a_2 \cos 2\theta$	-0.16667	-0.14434	-0.08334	0	+0.08334	+0.14434	+0.16667	+0.14434	+0.08334	0	-0.08334	-0.14434	-0.16667
$a_3 \cos 3\theta$	+0.07775	+0.05498	0	-0.05498	-0.07775	-0.05498	0	+0.05498	+0.07775	+0.05498	0	-0.05498	-0.07775
$a_4 \cos 4\theta$	+0.02083	+0.01042	-0.01042	-0.02083	-0.01042	+0.01042	+0.02083	+0.01042	-0.01042	-0.02083	-0.01042	+0.01042	+0.02083
$a_6 \cos 6\theta$	+0.00926	0	-0.00926	0	+0.00926	0	-0.00926	0	+0.00926	0	-0.00926	0	+0.00926
$\frac{\omega}{R}$	0.9812	0.9373	0.8433	0.7593	0.6946	0.6212	0.5185	0.3689	0.1503	-0.1204	-0.3688	-0.5246	-0.5738

Amp. = 1.5550868

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$$x = \frac{\pi}{12}\frac{R}{m}$$

$$\frac{w}{R} = 0.2477500 + 0.6836958 \cos\frac{m\psi}{R} - 0.1331667 \cos\frac{2m\psi}{R} + 0.0472106 \cos\frac{3m\psi}{R} + 0.0208333 \cos\frac{4m\psi}{R} + 0.009259259 \cos\frac{6m\psi}{R}$$

	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$a_1 \cos\theta$	0.68370	0.66039	0.59208	0.48344	0.34185	0.17694	0	-0.17694	-0.34185	-0.48344	-0.59208	-0.66039	-0.68370
$a_2 \cos 2\theta$	-0.13317	-0.11532	-0.06659	0	+0.06659	+0.11532	+0.13317	+0.11532	+0.06659	0	-0.06659	-0.11532	+0.13317
$a_3 \cos 3\theta$	+0.04721	+0.03338	0	-0.03338	-0.04721	-0.03338	0	+0.03338	+0.04721	+0.03338	0	-0.03338	-0.04721
$a_4 \cos 4\theta$													
$a_6 \cos 6\theta$													
	0.8756	0.8366	0.7536	0.6770	0.6078	0.5171	0.3925	0.2299	0.0185	-0.2231	-0.4306	-0.5509	-0.5862

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$$x = \frac{\pi}{6}\frac{R}{m}$$

$$\frac{w}{R} = 0.0659722 + 0.6320156\cos\frac{my}{R} - 0.0416167\cos\frac{2my}{R} - 0.03002109\cos\frac{3my}{R} + 0.0208333\cos\frac{4my}{R} + 0.00925925\cos\frac{6my}{R}$$

	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\cos\theta$	0.63202	0.61047	0.54733	0.44690	0.31601	0.16357	0	-0.16357	-0.31601	-0.44690	-0.54733	-0.61047	-0.63202
$\cos 2\theta$	-0.04167	-0.03609	-0.02084	0	+0.02084	+0.03609	+0.04167	+0.03609	+0.02084	0	-0.02084	-0.03609	-0.04167
$\cos 3\theta$	-0.03002	-0.02123	0	+0.02123	+0.03002	+0.02123	0	-0.02123	-0.03002	-0.02123	0	+0.02123	+0.03002
$\cos 4\theta$													
$\cos 6\theta$													
	0.6564	0.6295	0.5728	0.5133	0.4317	0.2973	0.1192	-0.0723	-0.2604	-0.4230	-0.5219	-0.5489	-0.5476

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$$x = \frac{\pi}{4}\frac{R}{m}$$

$$\frac{w}{R} = -0.0625000 + 0.5372767 \cos\frac{my}{R} + 0.0833333 \cos\frac{2my}{R} - 0.0974359 \cos\frac{3my}{R} + 0.0208333 \cos\frac{4my}{R} + 0.009259259 \cos\frac{6my}{R}$$

	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$a_1 \cos\theta$	0.53728	0.51896	0.46528	0.37991	0.26864	0.13905	0	-0.13905	-0.26864	-0.37991	-0.46528	-0.51896	-0.53728
$a_2 \cos 2\theta$	+0.08333	+0.07216	0.04167	0	-0.04167	-0.07216	-0.08333	-0.07216	-0.04167	0	+0.04167	+0.07216	+0.08333
$a_3 \cos 3\theta$	-0.09744	-0.06890	0	+0.06890	+0.09744	+0.06890	0	-0.06890	-0.09744	-0.06890	0	+0.06890	+0.09744
	0.4908	0.4701	0.4248	0.3655	0.2608	0.0837	-0.1343	-0.3322	-0.4714	-0.5321	-0.5058	-0.4300	-0.3889

$$x = \frac{\pi}{3}\frac{R}{m}$$

$$\frac{w}{R} = -0.1284722 + 0.3949262 \cos\frac{my}{R} + 0.2083333 \cos\frac{2my}{R} - 0.1227860 \cos\frac{3my}{R} + 0.0208333 \cos\frac{4my}{R} + 0.009259259 \cos\frac{6my}{R}$$

	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$a_1 \cos\theta$	0.39493	0.38146	0.34201	0.27926	0.19747	0.10221	0	-0.10221	-0.19747	-0.27926	-0.34201	-0.38146	-0.39493
$a_2 \cos 2\theta$	0.20833	0.18041	0.10417	0	-0.10417	-0.18041	-0.20833	-0.18041	-0.10417	0	+0.10417	+0.18041	+0.20833
$a_3 \cos 3\theta$	-0.12279	-0.08682	0	+0.08682	+0.12279	+0.08682	0	-0.08682	-0.12279	-0.08682	0	+0.08682	+0.12279
	0.3821	0.3570	0.2960	0.2468	0.0865	-0.1094	-0.3252	-0.4875	-0.5541	-0.5154	-0.3860	-0.2323	-0.1122

$$x = \frac{5\pi}{12}\frac{R}{m}$$

$$\frac{\Delta V}{R} = -0.185250 + 0.2101028 \cos\frac{m y}{R} + 0.2998333 \cos\frac{2 m y}{R} - 0.0839774 \cos\frac{3 m y}{R} + 0.0208333 \cos\frac{4 m y}{R} + 0.009259259 \cos\frac{6 m y}{R}$$

	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$a_1 \cos\theta$	0.2101	0.20294	0.18195	0.14856	0.10505	0.05437	0	-0.05437	-0.10505	-0.14856	-0.18195	-0.20294	-0.21010
$a_2 \cos 2\theta$	0.29983	0.25965	0.14992	0	-0.14992	-0.25965	-0.29983	-0.25965	-0.14992	0	+0.14992	+0.25965	+0.29983
$a_3 \cos 3\theta$	-0.08398	-0.05938	0	+0.05938	+0.08398	+0.05938	0	-0.05938	-0.08398	-0.05938	0	+0.05938	+0.08398
	0.2708	0.2284	0.1269	0.0019	-0.1473	-0.3207	-0.4735	-0.5482	-0.5254	-0.4140	-0.2420	-0.0567	+0.0186

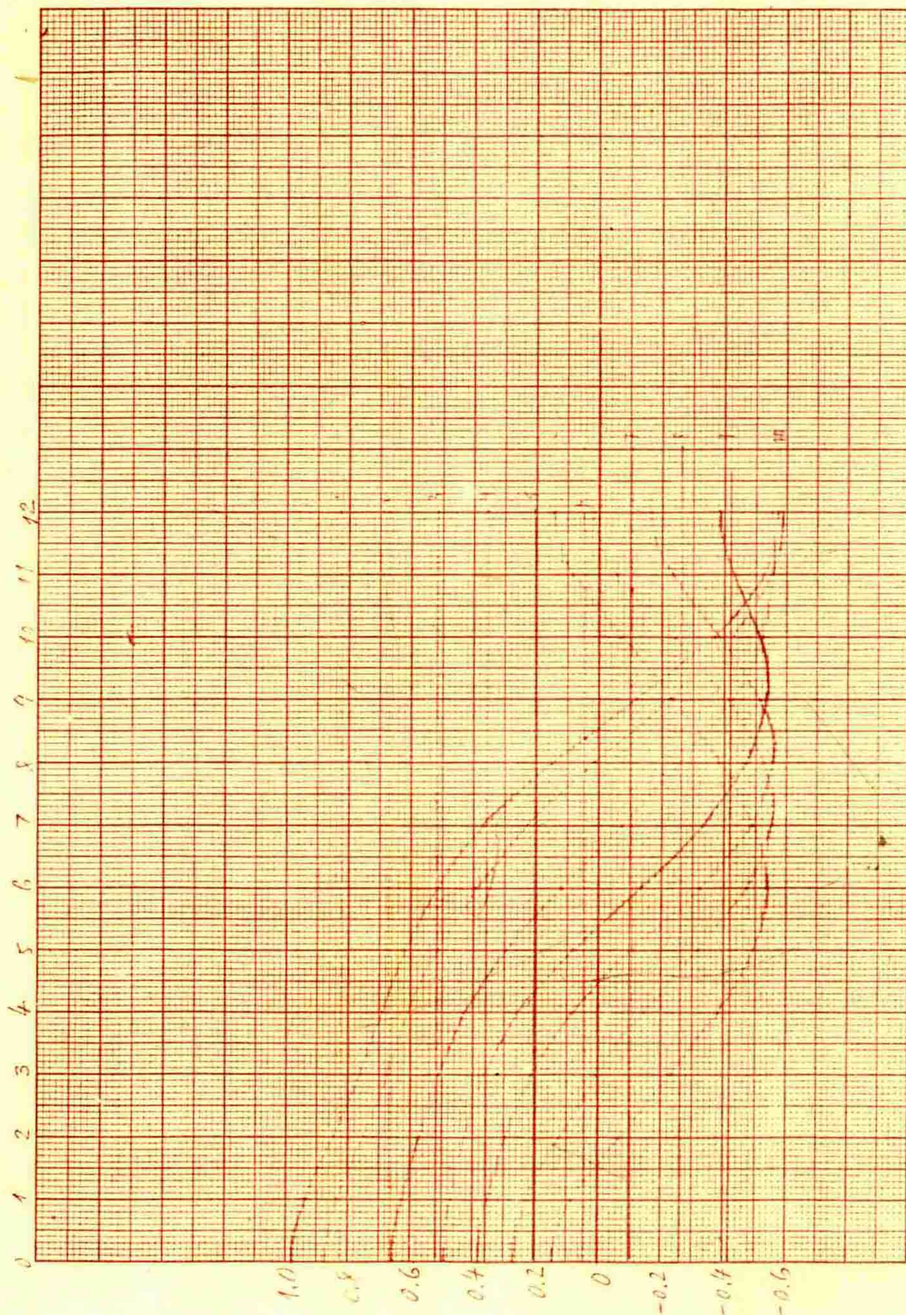
$$x = \frac{\pi}{2}\frac{R}{m}$$

$$\frac{\Delta V}{R} = -0.2152778 + 0.33333 \cos\frac{2 m y}{R} + 0.0208333 \cos\frac{4 m y}{R} + 0.009259259 \cos\frac{6 m x}{R}$$

	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$a_2 \cos 2\theta$	0.33333	0.28866	0.16667	0	-0.16667	-0.28866	-0.3333	-0.28866	-0.16667	0	+0.16667	+0.28866	+0.33333
	0.1481	0.0838	-0.0683	-0.2361	-0.3831	-0.49352	-0.5370						

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69)

$$\frac{w}{R} = f_0 + f_1 \Big\{ 0.160763 \cos\frac{2\pi x}{R} + 0.040191 \cos\frac{4\pi x}{R} + 0.017863 \cos\frac{6\pi x}{R}$$

$$+ 0.053588 \cos\frac{2\pi y}{R} + 0.013397 \cos\frac{4\pi y}{R} + 0.005954 \cos\frac{6\pi y}{R}$$

$$+ 0.469305 \cos\frac{\pi x}{R}\cos\frac{\pi y}{R} - 0.160763 \cos\frac{2\pi x}{R}\cos\frac{2\pi y}{R} - 0.019305 \cos\frac{3\pi x}{R}\cos\frac{\pi y}{R}$$

$$- 0.019305 \cos\frac{\pi x}{R}\cos\frac{3\pi y}{R} + 0.069305 \cos\frac{3\pi x}{R}\cos\frac{3\pi y}{R} \Big\} + \frac{f_2}{2}\Big\{ \cos\frac{\pi x}{R}\cos\frac{\pi y}{R} \Big\}$$

$$\frac{w}{R} = f_0 + 3af_1 \cos\frac{2\pi x}{R} + 3bf_1 \cos\frac{4\pi x}{R} + 3cf_1 \cos\frac{6\pi x}{R}$$

$$+ af_1 \cos\frac{2\pi y}{R} + bf_1 \cos\frac{4\pi y}{R} + cf_1 \cos\frac{6\pi y}{R}$$

$$+ (\alpha f_1 + 0.5 f_2) \cos\frac{\pi x}{R}\cos\frac{\pi y}{R} - \beta f_1 \cos\frac{2\pi x}{R}\cos\frac{2\pi y}{R} - \gamma f_1 \cos\frac{3\pi x}{R}\cos\frac{\pi y}{R}$$

$$- \gamma f_1 \cos\frac{\pi x}{R}\cos\frac{3\pi y}{R} + \delta f_1 \cos\frac{3\pi x}{R}\cos\frac{3\pi y}{R}$$

where

$a = 0.053588$, $b = 0.013397$, $c = 0.005954$

$\alpha = 0.469305$, $\beta = 0.160763$, $\gamma = 0.019305$

$\delta = 0.069305$

	a	b	c	α	β	γ	δ
a	0.0028717	0.00071792	0.00031906	0.0251491	0.0086149 7	0.0010345	0.0037139
b	0.00071792	0.00017948	0.00007977	0.00628728	0.00215374	0.00025863	0.00092848
c	0.00031906	0.00007977	0.00003545	0.00279424	0.00095718	0.00011494	0.00041264
α	0.0251491	0.00628728	0.00279424	0.2202472	0.07544688	0.00905993	0.03252518
β	0.00861497	0.00215374	0.00095718	0.07544688	0.02584474	0.00310353	0.01114168
γ	0.0010345	0.00025863	0.00011494	0.00905993	0.00310353	0.00037268	0.00133793
δ	0.0037139	0.00092848	0.00041264	0.03252518	0.01114168	0.00133793	0.00480318

$$\frac{\partial w}{\partial x} = -m\Big\{ 6af_1 \sin\frac{2mx}{R} + 12bf_1 \sin\frac{4mx}{R} + 18cf_1 \sin\frac{6mx}{R} + (\alpha f_1 + 0.5f_2)\sin\frac{mx}{R}\cos\frac{my}{R} - 2\beta f_1 \sin\frac{2mx}{R}\cos\frac{2my}{R}$$

$$- 3\gamma f_1 \sin\frac{3mx}{R}\cos\frac{my}{R} - \gamma f_1 \sin\frac{mx}{R}\cos\frac{3my}{R} + 3\delta f_1 \sin\frac{3mx}{R}\cos\frac{3my}{R}\Big\}$$

$$\frac{\partial w}{\partial y} = -m\Big\{ 2af_1 \sin\frac{2my}{R} + 4bf_1 \sin\frac{4my}{R} + 6cf_1 \sin\frac{6my}{R} + (\alpha f_1 + 0.5f_2)\cos\frac{mx}{R}\sin\frac{my}{R} - 2\beta f_1 \cos\frac{2mx}{R}\sin\frac{2my}{R}$$

$$- \gamma f_1 \cos\frac{3mx}{R}\sin\frac{my}{R} - 3\gamma f_1 \cos\frac{mx}{R}\sin\frac{3my}{R} + 3\delta f_1 \cos\frac{3mx}{R}\sin\frac{3my}{R}\Big\}$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{m^2}{R}\Big\{ 12af_1 \cos\frac{2mx}{R} + 48bf_1 \cos\frac{4mx}{R} + 108cf_1 \cos\frac{6mx}{R} + (\alpha f_1 + 0.5f_2)\cos\frac{mx}{R}\cos\frac{my}{R} - 4\beta f_1 \cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$- 9\gamma f_1 \cos\frac{3mx}{R}\cos\frac{my}{R} - \gamma f_1 \cos\frac{mx}{R}\cos\frac{3my}{R} + 9\delta f_1 \cos\frac{3mx}{R}\cos\frac{3my}{R}\Big\}$$

$$\frac{\partial^2 w}{\partial y^2} = -\frac{m^2}{R}\Big\{ 4af_1 \cos\frac{2my}{R} + 16bf_1 \cos\frac{4my}{R} + 36cf_1 \cos\frac{6my}{R} + (\alpha f_1 + 0.5f_2)\cos\frac{mx}{R}\cos\frac{my}{R} - 4\beta f_1 \cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$- \gamma f_1 \cos\frac{3mx}{R}\cos\frac{my}{R} - 9\gamma f_1 \cos\frac{mx}{R}\cos\frac{3my}{R} + 9\delta f_1 \cos\frac{3mx}{R}\cos\frac{3my}{R}\Big\}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{m^2}{R}\Big\{ (\alpha f_1 + 0.5f_2)\sin\frac{mx}{R}\sin\frac{my}{R} - 4\beta f_1 \sin\frac{2mx}{R}\sin\frac{2my}{R} - 3\gamma f_1 \sin\frac{3mx}{R}\sin\frac{my}{R}$$

$$- 3\gamma f_1 \sin\frac{mx}{R}\sin\frac{3my}{R} + 9\delta f_1 \sin\frac{3mx}{R}\sin\frac{3my}{R}\Big\}$$

$$
\begin{aligned}
I = \frac{m^4}{R^2}\Big\{ & \frac{1}{4}(\alpha f_1 + 0.5 f_2)^2 \left(1 - \cos\frac{2m\pi x}{R}\right)\left(1 - \cos\frac{2m\pi y}{R}\right) + 4\beta^2 f_1^2 \left(1 - \cos\frac{4m\pi x}{R}\right)\left(1 - \cos\frac{4m\pi y}{R}\right) \\
& + \frac{9}{4}\gamma^2 f_1^2 \left(1 - \cos\frac{6m\pi x}{R}\right)\left(1 - \cos\frac{2m\pi y}{R}\right) + \frac{9}{4}\gamma^2 f_1^2 \left(1 - \cos\frac{2m\pi x}{R}\right)\left(1 - \cos\frac{6m\pi y}{R}\right) + \frac{81}{4}\delta^2 f_1^2 \left(1 - \cos\frac{6m\pi x}{R}\right)\left(1 - \cos\frac{6m\pi y}{R}\right) \\
& - 2\beta f_1 (\alpha f_1 + 0.5 f_2)\left(\cos\frac{m\pi x}{R} - \cos\frac{3m\pi x}{R}\right)\left(\cos\frac{m\pi y}{R} - \cos\frac{3m\pi y}{R}\right) - \frac{3}{2}\gamma f_1 (\alpha f_1 + 0.5 f_2)\left(\cos\frac{2m\pi x}{R} - \cos\frac{4m\pi x}{R}\right)\left(1 - \cos\frac{2m\pi y}{R}\right) \\
& - \frac{3}{2}\gamma f_1 (\alpha f_1 + 0.5 f_2)\left(1 - \cos\frac{2m\pi x}{R}\right)\left(\cos\frac{2m\pi y}{R} - \cos\frac{4m\pi y}{R}\right) + \frac{9}{2}\delta f_1 (\alpha f_1 + 0.5 f_2)\left(\cos\frac{2m\pi x}{R} - \cos\frac{4m\pi x}{R}\right)\left(\cos\frac{2m\pi y}{R} - \cos\frac{4m\pi y}{R}\right) \\
& + 6\beta\gamma f_1^2 \left(\cos\frac{m\pi x}{R} - \cos\frac{5m\pi x}{R}\right)\left(\cos\frac{m\pi y}{R} - \cos\frac{3m\pi y}{R}\right) + 6\beta\gamma f_1^2 \left(\cos\frac{m\pi x}{R} - \cos\frac{3m\pi x}{R}\right)\left(\cos\frac{m\pi y}{R} - \cos\frac{5m\pi y}{R}\right) \\
& - 18\beta\delta f_1^2 \left(\cos\frac{m\pi x}{R} - \cos\frac{5m\pi x}{R}\right)\left(\cos\frac{m\pi y}{R} - \cos\frac{5m\pi y}{R}\right) + \frac{9}{2}\gamma^2 f_1^2 \left(\cos\frac{2m\pi x}{R} - \cos\frac{4m\pi x}{R}\right)\left(\cos\frac{2m\pi y}{R} - \cos\frac{4m\pi y}{R}\right) \\
& - \frac{27}{2}\gamma\delta f_1^2 \left(1 - \cos\frac{6m\pi x}{R}\right)\left(\cos\frac{2m\pi y}{R} - \cos\frac{4m\pi y}{R}\right) - \frac{27}{2}\gamma\delta f_1^2 \left(\cos\frac{2m\pi x}{R} - \cos\frac{4m\pi x}{R}\right)\left(1 - \cos\frac{6m\pi y}{R}\right)
\end{aligned}
$$

$$-\frac{1}{4}(\alpha f_1+0.5f_2)^2\left(1+\cos\frac{2\pi x}{R}\right)\left(1+\cos\frac{2\pi y}{R}\right)-4\beta^2 f_1^2\left(1+\cos\frac{4\pi x}{R}\right)\left(1+\cos\frac{4\pi y}{R}\right)-\frac{9}{4}\gamma^2 f_1^2\left(1+\cos\frac{6\pi x}{R}\right)\left(1+\cos\frac{2\pi y}{R}\right)$$

$$-\frac{9}{4}\gamma^2 f_1^2\left(1+\cos\frac{2\pi x}{R}\right)\left(1+\cos\frac{6\pi y}{R}\right)-\frac{81}{4}\delta^2 f_1^2\left(1+\cos\frac{6\pi x}{R}\right)\left(1+\cos\frac{6\pi y}{R}\right)$$

$$+2\beta f_1(\alpha f_1+0.5f_2)\left(\cos\frac{\pi x}{R}+\cos\frac{3\pi x}{R}\right)\left(\cos\frac{\pi y}{R}+\cos\frac{3\pi y}{R}\right)+\frac{5}{2}\gamma f_1(\alpha f_1+0.5f_2)\left(\cos\frac{2\pi x}{R}+\cos\frac{4\pi x}{R}\right)\left(1+\cos\frac{2\pi y}{R}\right)$$

$$+\frac{5}{2}\gamma f_1(\alpha f_1+0.5f_2)\left(1+\cos\frac{2\pi x}{R}\right)\left(\cos\frac{2\pi y}{R}+\cos\frac{4\pi y}{R}\right)-\frac{9}{2}\delta f_1(\alpha f_1+0.5f_2)\left(\cos\frac{2\pi x}{R}+\cos\frac{4\pi x}{R}\right)\left(\cos\frac{2\pi y}{R}+\cos\frac{4\pi y}{R}\right)$$

$$-10\beta\gamma f_1^2\left(\cos\frac{\pi x}{R}+\cos\frac{5\pi x}{R}\right)\left(\cos\frac{\pi y}{R}+\cos\frac{3\pi y}{R}\right)-10\beta\gamma f_1^2\left(\cos\frac{\pi x}{R}+\cos\frac{3\pi x}{R}\right)\left(\cos\frac{\pi y}{R}+\cos\frac{5\pi y}{R}\right)$$

$$+18\beta\delta f_1^2\left(\cos\frac{\pi x}{R}+\cos\frac{5\pi x}{R}\right)\left(\cos\frac{\pi y}{R}+\cos\frac{5\pi y}{R}\right)-\frac{41}{2}\gamma^2 f_1^2\left(\cos\frac{2\pi x}{R}+\cos\frac{4\pi x}{R}\right)\left(\cos\frac{2\pi y}{R}+\cos\frac{4\pi y}{R}\right)$$

$$+\frac{45}{2}\gamma\delta f_1^2\left(1+\cos\frac{6\pi x}{R}\right)\left(\cos\frac{2\pi y}{R}+\cos\frac{4\pi y}{R}\right)+\frac{45}{2}\gamma\delta f_1^2\left(\cos\frac{2\pi x}{R}+\cos\frac{4\pi x}{R}\right)\left(1+\cos\frac{6\pi y}{R}\right)$$

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$$
\begin{aligned}
I = -\frac{m^4}{R^2}\Bigg\{ & \frac{1}{2}(\alpha f_1+0.5f_2)^2\left(\cos\frac{2mx}{R}+\cos\frac{2my}{R}\right)+8\beta^2 f_1^2\left(\cos\frac{4mx}{R}+\cos\frac{4my}{R}\right)+\frac{9}{2}\gamma^2 f_1^2\left(\cos\frac{6mx}{R}+\cos\frac{6my}{R}\right)\\
&+\frac{9}{2}\gamma^2 f_1^2\left(\cos\frac{2mx}{R}+\cos\frac{6my}{R}\right)+\frac{81}{2}\delta^2 f_1^2\left(\cos\frac{6mx}{R}+\cos\frac{6my}{R}\right)-4\beta f_1(\alpha f_1+0.5f_2)\left(\cos\frac{3mx}{R}\cos\frac{my}{R}+\cos\frac{mx}{R}\cos\frac{3my}{R}\right)\\
&-\gamma f_1(\alpha f_1+0.5f_2)\left(\cos\frac{2mx}{R}+4\cos\frac{4mx}{R}+4\cos\frac{2mx}{R}\cos\frac{2my}{R}+\cos\frac{4mx}{R}\cos\frac{2my}{R}\right)\\
&-\gamma f_1(\alpha f_1+0.5f_2)\left(\cos\frac{2my}{R}+4\cos\frac{4my}{R}+4\cos\frac{2mx}{R}\cos\frac{2my}{R}+\cos\frac{2mx}{R}\cos\frac{4my}{R}\right)\\
&+9\delta f_1(\alpha f_1+0.5f_2)\left(\cos\frac{4mx}{R}\cos\frac{2my}{R}+\cos\frac{2mx}{R}\cos\frac{4my}{R}\right)+4\beta\gamma f_1^2\left(\cos\frac{mx}{R}\cos\frac{my}{R}+4\cos\frac{5mx}{R}\cos\frac{my}{R}\right.\\
&\left.+4\cos\frac{mx}{R}\cos\frac{3my}{R}+\cos\frac{5mx}{R}\cos\frac{3my}{R}\right)+4\beta\gamma f_1^2\left(\cos\frac{mx}{R}\cos\frac{my}{R}+4\cos\frac{mx}{R}\cos\frac{5my}{R}+4\cos\frac{3mx}{R}\cos\frac{my}{R}\right.\\
&\left.+\cos\frac{3mx}{R}\cos\frac{5my}{R}\right)-36\beta\delta f_1^2\left(\cos\frac{5mx}{R}\cos\frac{my}{R}+\cos\frac{mx}{R}\cos\frac{5my}{R}\right)+\gamma^2 f_1^2\left(16\cos\frac{2mx}{R}\cos\frac{2my}{R}+25\cos\frac{4mx}{R}\cos\frac{2my}{R}\right.\\
&\left.+25\cos\frac{2mx}{R}\cos\frac{4my}{R}+16\cos\frac{4mx}{R}\cos\frac{4my}{R}\right)-9\gamma\delta f_1^2\left(\cos\frac{2my}{R}+4\cos\frac{6mx}{R}\cos\frac{2my}{R}+4\cos\frac{4my}{R}+\cos\frac{6mx}{R}\cos\frac{4my}{R}\right)\\
&-9\gamma\delta f_1^2\left(\cos\frac{2mx}{R}+4\cos\frac{2mx}{R}\cos\frac{6my}{R}+4\cos\frac{4mx}{R}+\cos\frac{4mx}{R}\cos\frac{6my}{R}\right)\Bigg\}
\end{aligned}
$$

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$$II = -\frac{m^4}{R^2}\Big\{ 48a^2f_1^2\cos\frac{2mx}{R}\cos\frac{2my}{R} + 192abf_1^2\cos\frac{4mx}{R}\cos\frac{2my}{R} + 432acf_1^2\cos\frac{6mx}{R}\cos\frac{2my}{R}$$

$$+ 2af_1(\alpha f_1 + 0.5f_2)\left(\cos\frac{mx}{R}\cos\frac{my}{R} + \cos\frac{mx}{R}\cos\frac{3my}{R}\right) - 8a\beta f_1^2\left(\cos^2\frac{2mx}{R} + \cos\frac{2mx}{R}\cos\frac{4my}{R}\right)$$

$$- 18a\gamma f_1^2\left(\cos\frac{3mx}{R}\cos\frac{my}{R} + \cos\frac{3mx}{R}\cos\frac{3my}{R}\right) - 9a\gamma f_1^2\left(\cos\frac{mx}{R}\cos\frac{my}{R} + \cos\frac{mx}{R}\cos\frac{5my}{R}\right)$$

$$+ 18a\delta f_1^2\left(\cos\frac{3mx}{R}\cos\frac{my}{R} + \cos\frac{3mx}{R}\cos\frac{5my}{R}\right) + 192abf_1^2\cos\frac{2mx}{R}\cos\frac{4my}{R} + 768b^2f_1^2\cos\frac{4mx}{R}\cos\frac{4my}{R}$$

$$+ 1728bcf_1^2\cos\frac{6mx}{R}\cos\frac{4my}{R} + 8bf_1(\alpha f_1 + 0.5f_2)\left(\cos\frac{mx}{R}\cos\frac{3my}{R} + \cos\frac{mx}{R}\cos\frac{5my}{R}\right)$$

$$- 32b\beta f_1^2\left(\cos\frac{2mx}{R}\cos\frac{2my}{R} + \cos\frac{2mx}{R}\cos\frac{6my}{R}\right) - 72b\gamma f_1^2\left(\cos\frac{3mx}{R}\cos\frac{3my}{R} + \cos\frac{3mx}{R}\cos\frac{5my}{R}\right)$$

$$- 8b\gamma f_1^2\left(\cos\frac{mx}{R}\cos\frac{my}{R} + \cos\frac{mx}{R}\cos\frac{7my}{R}\right) + 72b\delta f_1^2\left(\cos\frac{3mx}{R}\cos\frac{my}{R} + \cos\frac{3mx}{R}\cos\frac{7my}{R}\right)$$

$$+ 342acf_1^2\cos\frac{2mx}{R}\cos\frac{6my}{R} + 1728bcf_1^2\cos\frac{4mx}{R}\cos\frac{6my}{R} + 3888c^2f_1^2\cos\frac{6mx}{R}\cos\frac{6my}{R}$$

$$+ 18cf_1(\alpha f_1 + 0.5f_2)\left(\cos\frac{mx}{R}\cos\frac{5my}{R} + \cos\frac{mx}{R}\cos\frac{7my}{R}\right) - 72c\beta f_1^2\left(\cos\frac{2mx}{R}\cos\frac{4my}{R} + \cos\frac{2mx}{R}\cos\frac{8my}{R}\right)$$

$$- 162c\gamma f_1^2\left(\cos\frac{3mx}{R}\cos\frac{5my}{R} + \cos\frac{3mx}{R}\cos\frac{7my}{R}\right) - 18c\gamma f_1^2\left(\cos\frac{mx}{R}\cos\frac{3my}{R} + \cos\frac{mx}{R}\cos\frac{9my}{R}\right)$$

$$+ 162c\delta f_1^2\left(\cos\frac{3mx}{R}\cos\frac{3my}{R} + \cos\frac{3mx}{R}\cos\frac{9my}{R}\right)$$

Cont'd !!!

Cont'd !!!

$$+ 6af_1(\alpha f_1 + 0.5f_2)\left(\cos\frac{\pi x}{R}\cos\frac{\pi y}{R} + \cos\frac{3\pi x}{R}\cos\frac{\pi y}{R}\right) + 24bf_1(\alpha f_1 + 0.5f_2)\left(\cos\frac{3\pi x}{R}\cos\frac{\pi y}{R} + \cos\frac{5\pi x}{R}\cos\frac{\pi y}{R}\right)$$

$$+ 54cf_1(\alpha f_1 + 0.5f_2)\left(\cos\frac{5\pi x}{R}\cos\frac{\pi y}{R} + \cos\frac{7\pi x}{R}\cos\frac{\pi y}{R}\right) - 24a\beta f_1^2\left(\cos\frac{2\pi y}{R} + \cos\frac{4\pi x}{R}\cos\frac{2\pi y}{R}\right)$$

$$- 96b\beta f_1^2\left(\cos\frac{2\pi x}{R}\cos\frac{2\pi y}{R} + \cos\frac{6\pi x}{R}\cos\frac{2\pi y}{R}\right) - 216c\beta f_1^2\left(\cos\frac{4\pi x}{R}\cos\frac{2\pi y}{R} + \cos\frac{8\pi x}{R}\cos\frac{2\pi y}{R}\right)$$

$$- 6a\gamma f_1^2\left(\cos\frac{\pi x}{R}\cos\frac{\pi y}{R} + \cos\frac{5\pi x}{R}\cos\frac{\pi y}{R}\right) - 24b\gamma f_1^2\left(\cos\frac{\pi x}{R}\cos\frac{\pi y}{R} + \cos\frac{7\pi x}{R}\cos\frac{\pi y}{R}\right)$$

$$- 54c\gamma f_1^2\left(\cos\frac{3\pi x}{R}\cos\frac{\pi y}{R} + \cos\frac{9\pi x}{R}\cos\frac{\pi y}{R}\right) - 54a\gamma f_1^2\left(\cos\frac{\pi x}{R}\cos\frac{3\pi y}{R} + \cos\frac{3\pi x}{R}\cos\frac{3\pi y}{R}\right)$$

$$- 216b\gamma f_1^2\left(\cos\frac{3\pi x}{R}\cos\frac{3\pi y}{R} + \cos\frac{5\pi x}{R}\cos\frac{3\pi y}{R}\right) - 486c\gamma f_1^2\left(\cos\frac{5\pi x}{R}\cos\frac{3\pi y}{R} + \cos\frac{7\pi x}{R}\cos\frac{3\pi y}{R}\right)$$

$$+ 54a\delta f_1^2\left(\cos\frac{\pi x}{R}\cos\frac{3\pi y}{R} + \cos\frac{5\pi x}{R}\cos\frac{3\pi y}{R}\right) + 216b\delta f_1^2\left(\cos\frac{\pi x}{R}\cos\frac{3\pi y}{R} + \cos\frac{7\pi x}{R}\cos\frac{3\pi y}{R}\right)$$

$$+ 486c\delta f_1^2\left(\cos\frac{3\pi x}{R}\cos\frac{3\pi y}{R} + \cos\frac{9\pi x}{R}\cos\frac{3\pi y}{R}\right)\Bigg\}$$

$$III = +\frac{m^2}{R^2}\Big\{ 12af_1\cos\frac{2mx}{R} + 48bf_1\cos\frac{4mx}{R} + 108cf_1\cos\frac{6mx}{R} + (\alpha f_1 + 0.5f_2)\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$-4\beta f_1\cos\frac{2mx}{R}\cos\frac{2my}{R} - 9\gamma f_1\cos\frac{3mx}{R}\cos\frac{my}{R} - \gamma f_1\cos\frac{mx}{R}\cos\frac{3my}{R} + 9\delta f_1\cos\frac{3mx}{R}\cos\frac{3my}{R}\Big\}$$

1 $\boxed{\cos\frac{2mx}{R}}$: $m^2\Big\{\frac{1}{2}(\alpha f_1+0.5f_2)^2 + \frac{9}{2}\gamma^2 f_1^2 - \gamma f_1(\alpha f_1+0.5f_2) - 9\gamma\delta f_1^2 - 8a\beta f_1^2\Big\} - 12af_1$

2 $\boxed{\cos\frac{4mx}{R}}$ $m^2\Big\{8\beta^2 f_1^2 - 4\gamma f_1(\alpha f_1+0.5f_2) - 36\gamma\delta f_1^2\Big\} - 48bf_1$

3 $\boxed{\cos\frac{6mx}{R}}$: $m^2\Big\{\frac{9}{2}\gamma^2 f_1^2 + \frac{81}{2}\delta^2 f_1^2\Big\} - 108cf_1$

4 $\boxed{\cos\frac{2my}{R}}$: $m^2\Big\{\frac{1}{2}(\alpha f_1+0.5f_2)^2 + \frac{9}{2}\gamma^2 f_1^2 - \gamma f_1(\alpha f_1+0.5f_2) - 9\gamma\delta f_1^2 - 24a\beta f_1^2\Big\}$

5 $\boxed{\cos\frac{4my}{R}}$: $m^2\Big\{8\beta^2 f_1^2 - 4\gamma f_1(\alpha f_1+0.5f_2) - 36\gamma\delta f_1^2\Big\}$

6 $\boxed{\cos\frac{6my}{R}}$: $m^2\Big\{\frac{9}{2}\gamma^2 f_1^2 + \frac{81}{2}\delta^2 f_1^2\Big\}$

7 $\boxed{\cos\frac{mx}{R}\cos\frac{my}{R}}$ $m^2\Big\{4\beta\gamma f_1^2 + 4\beta\gamma f_1^2 + 2af_1(\alpha f_1+0.5f_2) - 2a\gamma f_1^2 - 8b\gamma f_1^2 + 6af_1(\alpha f_1+0.5f_2)$

$$-6a\gamma f_1^2 - 24b\gamma f_1^2\Big\} - (\alpha f_1 + 0.5f_2)$$

8 $\boxed{\cos\frac{2\pi x}{R}\cos\frac{2\pi y}{R}}$: $m^2\Big\{-4\gamma f_1(\alpha f_1+0.5f_2)-4\gamma f_1(\alpha f_1+0.5f_2)+16\gamma^2 f_1^2+48a^2 f_1^2-32b\beta f_1^2$

$-96b\beta f_1^2\Big\}+4\beta f_1$

9 $\boxed{\cos\frac{3\pi x}{R}\cos\frac{\pi y}{R}}$: $m^2\Big\{-4\beta f_1(\alpha f_1+0.5f_2)+16\beta\gamma f_1^2-18a\gamma f_1^2+18a\delta f_1^2+72b\delta f_1^2+6af_1(\alpha f_1+0.5f_2)$

$+24bf_1(\alpha f_1+0.5f_2)-54c\gamma f_1^2\Big\}+9\gamma f_1$

10 $\boxed{\cos\frac{\pi x}{R}\cos\frac{3\pi y}{R}}$: $m^2\Big\{-4\beta f_1(\alpha f_1+0.5f_2)+16\beta\gamma f_1^2+2af_1(\alpha f_1+0.5f_2)+8bf_1(\alpha f_1+0.5f_2)-18c\gamma f_1^2$

$-54a\gamma f_1^2+54a\delta f_1^2+216b\delta f_1^2\Big\}+\gamma f_1$

11 $\boxed{\cos\frac{3\pi x}{R}\cos\frac{3\pi y}{R}}$: $m^2\Big\{-18a\gamma f_1^2-72b\gamma f_1^2+162c\delta f_1^2-54a\gamma f_1^2-216b\gamma f_1^2+486c\delta f_1^2\Big\}+9\delta f_1$

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12) $\cos\frac{2m\pi x}{R}\cos\frac{5m\pi y}{R}$: $m^2\left\{-36\beta\delta f_1^2 - 2a\gamma f_1^2 + 8bf_1(\alpha f_1 + 0.5f_2) + 18cf_1(\alpha f_1 + 0.5f_2) + 16\beta\gamma f_1^2\right\}$

13) $\cos\frac{5m\pi x}{R}\cos\frac{m\pi y}{R}$: $m^2\left\{16\beta\gamma f_1^2 - 36\beta\delta f_1^2 + 24bf_1(\alpha f_1 + 0.5f_2) + 54cf_1(\alpha f_1 + 0.5f_2) - 6a\gamma f_1^2\right\}$

14) $\cos\frac{2m\pi x}{R}\cos\frac{4m\pi y}{R}$: $m^2\left\{-\gamma f_1(\alpha f_1 + 0.5f_2) + 9\delta f_1(\alpha f_1 + 0.5f_2) + 25\gamma^2 f_1^2 - 8a\beta f_1^2 + 192abf_1^2 - 72c\beta f_1^2\right\}$

15) $\cos\frac{4m\pi x}{R}\cos\frac{2m\pi y}{R}$: $m^2\left\{-\gamma f_1(\alpha f_1 + 0.5f_2) + 9\delta f_1(\alpha f_1 + 0.5f_2) + 192abf_1^2 - 24a\beta f_1^2 - 216c\beta f_1^2 + 25\gamma^2 f_1^2\right\}$

16) $\cos\frac{m\pi x}{R}\cos\frac{7m\pi y}{R}$: $m^2\left\{-8b\gamma f_1^2 + 18cf_1(\alpha f_1 + 0.5f_2)\right\}$

17) $\cos\frac{7m\pi x}{R}\cos\frac{m\pi y}{R}$: $m^2\left\{54cf_1(\alpha f_1 + 0.5f_2) - 24b\gamma f_1^2\right\}$

18) $\cos\frac{2m\pi x}{R}\cos\frac{6m\pi y}{R}$: $m^2\left\{-36\gamma\delta f_1^2 - 32b\beta f_1^2 + 342acf_1^2\right\}$

19) $\cos\frac{6m\pi x}{R}\cos\frac{2m\pi y}{R}$: $m^2\left\{-36\gamma\delta f_1^2 + 432acf_1^2 - 96b\beta f_1^2\right\}$

20 $\boxed{\cos\frac{3\pi x}{R}\cos\frac{5\pi y}{R}}$: $m^2\left\{4\beta\gamma f_1^2+18a\delta f_1^2-72b\gamma f_1^2-162c\gamma f_1^2\right\}$

21 $\boxed{\cos\frac{5\pi x}{R}\cos\frac{3\pi y}{R}}$: $m^2\left\{4\beta\gamma f_1^2-216b\gamma f_1^2+54a\delta f_1^2-486c\gamma f_1^2\right\}$

22 $\boxed{\cos\frac{4\pi x}{R}\cos\frac{4\pi y}{R}}$: $m^2\left\{16\gamma^2 f_1^2+768b^2 f_1^2\right\}$

23 $\boxed{\cos\frac{\pi x}{R}\cos\frac{9\pi y}{R}}$ $m^2\left\{-18c\gamma f_1^2\right\}$

24 $\boxed{\cos\frac{9\pi x}{R}\cos\frac{\pi y}{R}}$ $m^2\left\{-54c\gamma f_1^2\right\}$

25 $\boxed{\cos\frac{4\pi x}{R}\cos\frac{6\pi x}{R}}$ $m^2\left\{-9\gamma\delta f_1^2+1728bc f_1^2\right\}$

26 $\boxed{\cos\frac{6\pi x}{R}\cos\frac{4\pi y}{R}}$ $m^2\left\{-9\gamma\delta f_1^2+1728bc f_1^2\right\}$

27 $\boxed{\cos\frac{3\pi x}{R}\cos\frac{7\pi y}{R}}$ $m^2\left\{72b\delta f_1^2-162c\gamma f_1^2\right\}$

28 $\boxed{\cos\frac{7\pi x}{R}\cos\frac{3\pi y}{R}}$ $m^2\left\{-486c\gamma f_1^2+216b\delta f_1^2\right\}$

29 $\boxed{\cos\frac{2\pi x}{R}\cos\frac{7\pi y}{R}}$ $m^2\left\{-72c\beta f_1^2\right\}$

30 $\boxed{\cos\frac{7\pi x}{R}\cos\frac{2\pi y}{R}}$ $m^2\left\{-216c\beta f_1^2\right\}$

31 $\boxed{\cos\frac{6\pi x}{R}\cos\frac{6\pi y}{R}}$ $m^2\left\{3888c^2 f_1^2\right\}$

32 $\boxed{\cos\frac{3\pi x}{R}\cos\frac{9\pi y}{R}}$ $m^2\left\{162c\delta f_1^2\right\}$

33 $\boxed{\cos\frac{9\pi x}{R}\cos\frac{3\pi y}{R}}$ $m^2\left\{486c\delta f_1^2\right\}$

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$$1 = m^2\left\{ f_1^2\left(\tfrac{1}{2}\alpha^2 + \tfrac{9}{2}\gamma^2 - \alpha\gamma - 9\gamma\delta - 8a\beta\right) + f_1f_2\left(\tfrac{1}{2}\alpha - 0.5\gamma\right) + \tfrac{1}{8}f_2^2 \right\} - 12af_1$$

$$= m^2\left\{ 0.0477796\, f_1^2 + 0.225\, f_1f_2 + 0.125000\, f_2^2 \right\} - 0.643056\, f_1 \qquad \left(\tfrac{1}{8}\right)$$

$$2 = m^2\left\{ f_1^2\left(8\beta^2 - 4\alpha\gamma - 36\gamma\delta\right) - f_1f_2\left(2\gamma\right) \right\} - 48bf_1$$

$$= m^2\left\{ 0.122353\, f_1^2 - 0.038610\, f_1f_2 \right\} - 0.643056\, f_1 \qquad \left(\tfrac{1}{128}\right)$$

$$3 = m^2\left\{ 0.196206\, f_1^2 \right\} - 0.643056\, f_1 \qquad \left(\tfrac{1}{648}\right)$$

$$4 = m^2\left\{ \left(\tfrac{1}{2}\alpha^2 + \tfrac{9}{2}\gamma^2 - \alpha\gamma - 9\gamma\delta - 24a\beta\right) f_1^2 + \left(\tfrac{1}{2}\alpha - 0.5\gamma\right) f_1f_2 + \tfrac{1}{8} f_2^2 \right\}$$

$$= m^2\left\{ -0.1160599\, f_1^2 + 0.225\, f_1f_2 + 0.125000\, f_2^2 \right\} \qquad \left(\tfrac{1}{8}\right)$$

$$5 = m^2\left\{ 0.122353\, f_1^2 - 0.038610\, f_1f_2 \right\} \qquad \left(\tfrac{1}{128}\right)$$

$$6 = m^2\left\{ 0.196206\, f_1^2 \right\} \qquad \left(\tfrac{1}{648}\right)$$

$$7 = m^2\left\{ f_1^2\left(8\beta\gamma + 6a\alpha - 8a\gamma - 32b\beta\right) + 3af_1f_2 \right\} - \left(\alpha f_1 + 0.5f_2\right)$$

$$= m^2\left\{ 0.159171\, f_1^2 + 0.160764\, f_1f_2 \right\} - \left(0.469305\, f_1 + 0.5000\, f_2\right) \qquad \left(\tfrac{1}{4}\right)$$

$$8 = m^2\left\{(-8\alpha\gamma + 16\gamma^2 + 48a^2 - 108b\beta)f_1^2 - 4\gamma f_1 f_2\right\} + 4\beta f_1$$

$$= -\left[m^2\left\{0.161279 f_1^2 + 0.077220 f_1 f_2\right\} - 0.643052 f_1\right] \quad \left(\frac{1}{14}\right)$$

$$9 = m^2\left\{(-4\alpha\beta + 16\beta\gamma - 18a\gamma + 18a\delta + 72b\delta + 6a\alpha + 24b\alpha - 54c\gamma)f_1^2 + (-2\beta + 3a + 12b)f_1 f_2\right\} + 9\gamma f_1$$

$$= m^2\left\{0.158531 f_1^2 + 0.000002 f_1 f_2\right\} + 0.173745 f_1 \quad \left(\frac{1}{100}\right)$$

$$10 = m^2\left\{(-4\alpha\beta + 16\beta\gamma - 54a\gamma + 54a\delta + 216b\delta + 2a\alpha + 8b\alpha - 18c\gamma)f_1^2 + (-2\beta + a + 4b)f_1 f_2\right\} + \gamma f_1$$

$$= m^2\left\{0.191636 f_1^2 - 0.21435 f_1 f_2\right\} + 0.019305 f_1 \quad \left(\frac{1}{100}\right)$$

$$11 = m^2\left\{(-72a\gamma - 288b\gamma + 648c\delta)f_1^2\right\} + 9\delta f_1$$

$$= m^2\left\{0.118421 f_1^2\right\} + 0.623745 f_1 \quad \left(\frac{1}{324}\right)$$

$$12 = m^2\left\{ f_1^2(-36\beta\delta - 2a\gamma + 8b\alpha + 18c\alpha + 16\beta\gamma) + f_1f_2(4b + 9c)\right\}$$

$$= m^2\left\{ -0.252918 f_1^2 + 0.107174 f_1f_2 \right\} \qquad \left(\frac{1}{6+6}\right)$$

$$13 = m^2\left\{ f_1^2(-36\beta\delta - 6a\gamma + 24b\alpha + 54c\alpha + 16\beta\gamma) + f_1f_2(12b + 27c)\right\}$$

$$= m^2\left\{ -0.055867 f_1^2 + 0.321522 f_1f_2 \right\} \qquad \left(\frac{1}{6+1}\right)$$

$$14 = m^2\left\{ (-\alpha\gamma + 9\alpha\delta + 25\gamma^2 - 8a\beta + 192ab - 72c\beta)f_1^2 + f_1f_2(-0.5\gamma + 4.5\delta)\right\}$$

$$= m^2\left\{ 0.292988 f_1^2 + 0.302220 f_1f_2 \right\} \qquad \left(\frac{1}{400}\right)$$

$$15 = m^2\left\{ (-\alpha\gamma + 9\alpha\delta + 192ab - 24a\beta + 25\gamma^2 - 216c\beta)f_1^2 + f_1f_2(-0.5\gamma + 4.5\delta)\right\}$$

$$= m^2\left\{ (0.017314 f_1^2 + 0.3027\ f_1f_2)\right\} \qquad \left(\frac{1}{400}\right)$$

$$16 = m^2\left\{ 0.048227 f_1^2 + 0.053586 f_1f_2 \right\} \qquad \left(\frac{1}{2500}\right)$$

$$\Updownarrow$$

$$17 = m^2\left\{ 0.144681 f_1^2 + 0.160758 f_1f_2 \right\} \qquad \left(\frac{1}{2500}\right)$$

$18 = m^2\{-0.007967 f_1^2\} \quad \left(\frac{1}{1600}\right)$

$19 = m^2\{-0.117091 f_1^2\} \quad \left(\frac{1}{1600}\right)$

$20 = m^2\{0.042023 f_1^2\} \quad \left(\frac{1}{1156}\right)$

$21 = m^2\{0.101240 f_1^2\} \quad \left(\frac{1}{1156}\right)$

$22 = m^2\{0.143804 f_1^2\} \quad \left(\frac{1}{1024}\right)$

$23 = m^2\{-0.0020689 f_1^2\} \quad \left(\frac{1}{6724}\right)$

$24 = m^2\{-0.0062067 f_1^2\} \quad \left(\frac{1}{6724}\right)$

$25 = m^2\{0.125801 f_1^2\} \quad \left(\frac{1}{2704}\right)$

$26 = m^2\{0.125801 f_1^2\} \quad \left(\frac{1}{2704}\right)$

$27 = m^2\{0.048230 f_1^2\} \quad \left(\frac{1}{3364}\right)$

$28 = m^2\{0.144691 f_1^2\} \quad \left(\frac{1}{3364}\right)$

$29 = m^2\{-0.068917 f_1^2\} \quad \left(\frac{1}{4624}\right)$

$30 = m^2\{-0.201751 f_1^2\} \quad \left(\frac{1}{4624}\right)$

$31 = m^2\{0.137130 f_1^2\} \quad \left(\frac{1}{5184}\right)$

$32 = m^2\{0.066848 f_1^2\} \quad \left(\frac{1}{8100}\right)$

$33 = m^2\{0.200543 f_1^2\} \quad \left(\frac{1}{8100}\right)$

$$m^4 f_1^4 \{ 0.0000592939 + 0.0001169551 + 0.0000594086 + 0.0016837375 + 0.0001169551 + 0.0000594086$$
$$+ 0.0063338518 + 0.0004064206 + 0.0002513 2078 + 0.00036724356 + 0.0000432825$$
$$+ 0.0000946265 + 0.0000046170 + 0.0002146049 + 0.0000007494 + 0.0000009303$$
$$+ 0.0000083730 + 0.0000000397 + 0.0000085689 + 0.0000015276 + 0.0000088664$$
$$+ 0.0000201949 + 0.0000000064 + 0.0000117055 + 0.0000069147 + 0.0000102715$$
$$+ 0.0000036646 + 0.0000055169 \} = \underline{0.009899056\, m^4 f_1^4}$$

$$m^4 f_1^3 f_2 \{ 0.0012251025 - 0.0000738133 - 0.0065281444 - 0.0000738133 + 0.0127944833$$
$$+ 0.0003891864 + 0.0000000063 - 0.0008215435 - 0.0000801960 - 0.0000531434$$
$$+ 0.0004427342 + 0.0000261631 + 0.0000020674 + 0.0000186069 \}$$

$$= \underline{0.007267696\, m^4 f_1^3 f_2}$$

$$m^4 f_1^2 f_2^2 \{ 0.0063281250 + 0.0006806125 + 0.0000116463 + 0.0063281250 + 0.0006806125 + 0.0000116463$$
$$+ 0.0064612659 + 0.0000931708 + 0 + 0.0004574572 + 0.0000169915 + 0.0001529236$$
$$+ 0.0002213423 + 0.0002213423 + 0.0000114858 \}$$

$$= \underline{0.0216927 5\, m^4 f_1^2 f_2^2}$$

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$$\underline{+0.00703125\, m^4 f_1 f_2^3 + 0.001953125\, m^4 f_2^4}$$

$$- m^2 f_1^3 \{ 0.0035013756 + 0.0012293724 + 0.0003894180 + 0.0373498731 + 0.0032409620 - 0.0005507794$$

$$- 0.0000739907 - 0.0004559537 \}$$

$$= \underline{- 0.04463018\, m^2 f_1^3}$$

$$- m^2 f_1^2 f_2 \{ 0.0361719000 - 0.0003879436 + 0.0775164245 + 0.0015517649 - 0.0000000370$$

$$+ 0.0000827605 \}$$

$$= \underline{- 0.11493490\, m^2 f_1^2 f_2}$$

$$- m^2 f_1 f_2^2 \{ 0.0200955 + 0.0401910 \} = \underline{-0.06028650\, m^2 f_1 f_2^2}$$

$$+ f_1^2 \{ 0.0516901274 + 0.0032306330 + 0.0006361797 + 0.0550617958 + 0.0064611855$$

$$+ 0.0003018733 + 0.0000037268 + 0.0012007958 \} = \underline{0.11858829\, f_1^2}$$

$$\underline{0.11732625\, f_1 f_2}$$

$$\underline{0.06250000\, f_2^2}$$

$$\delta_1 = 0.009879056\, m^4 f_1^4 + 0.007267696\, m^4 f_1^3 f_2 + 0.02169275\, m^4 f_1^2 f_2^2 + 0.00703125\, m^4 f_1 f_2^3 + 0.001953125\, m^4 f_2^4$$

$$- 0.04463018\, m^2 f_1^3 - 0.11493490\, m^2 f_1^2 f_2 - 0.06027650\, m^2 f_1 f_2^2$$

$$+ 0.11158829\, f_1^2 + 0.11732625\, f_1 f_2 + 0.06250000\, f_2^2$$

---

the constant term in $\left(\frac{\partial w}{\partial x}\right)^2$ is

$$m^2 \left\{ 18a^2 f_1^2 + 72 b^2 f_1^2 + 162 c^2 f_1^2 + \frac{1}{4}(\alpha f_1 + 0.5 f_2)^2 + \beta^2 f_1^2 + \frac{9}{4}\gamma^2 f_1^2 + \frac{1}{4}\gamma^2 f_1^2 + \frac{9}{4}\delta^2 f_1^2 \right\}$$

$$K = -\frac{\sigma}{E} m^2 \left\{ f_1^2 (72a^2 + 288 b^2 + 648 c^2 + \alpha^2 + 4\beta^2 + 9\gamma^2 + \gamma^2 + 9\delta^2) + \alpha f_1 f_2 + \frac{1}{4} f_2^2 \right\}$$

$$\boxed{K = -\frac{\sigma}{E} m^2 \left\{ 0.6520058\, f_1^2 + 0.469305\, f_1 f_2 + 0.250000\, f_2^2 \right\}}$$

$$\beta_2 = \frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \Big\{ 288 a^2 f_1^2 + 4608 b^2 f_1^2 + 23328 c^2 f_1^2 + 32 a^2 f_1^2 + 512 b^2 f_1^2 + 2592 c^2 f_1^2$$

$$+ 4(\alpha f_1 + 0.5 f_2)^2 + 64 \beta^2 f_1^2 + 2 \times 100 \gamma^2 f_1^2 + 324 \delta^2 f_1^2 \Big\}$$

$$= \frac{1}{3(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \Big\{ f_1^2 \big(\overset{80}{\cancel{72}} a^2 + \overset{1280}{\cancel{1152}} b^2 + \overset{6480}{\cancel{5832}} c^2 + \cancel{8a^2} + \cancel{128 b^2} + \cancel{648 c^2} + \alpha^2 + 16\beta^2 + 50\gamma^2$$

$$+ 81\delta^2 \big) + \alpha f_1 f_2 + 0.25 f_2^2 \Big\}$$

$$\boxed{\beta_2 = \frac{1}{3(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \Big\{ 1.730641 f_1^2 + 0.469305 f_1 f_2 + 0.250000 f_2^2 \Big\}}$$

$$\frac{\sigma}{E} m^2 \Big\{ 1.3040116 f_1 + 0.469305 f_2 \Big\} = \frac{1}{3(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \Big\{ 3.461282 f_1 + 0.469305 f_2 \Big\}$$

$$+ m^4 \Big\{ 0.039596224 f_1^3 + 0.021803088 f_1^2 f_2 + 0.043385500 f_1 f_2^2 + 0.00703125 f_2^3 \Big\}$$

$$- m^2 \Big\{ 0.13389054 f_1^2 + 0.22986980 f_1 f_2 + 0.06028650 f_2^2 \Big\} + \Big\{ 0.23717658 f_1 + 0.11732625 f_2 \Big\}$$

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$$\frac{\sigma}{E} m^2 \left\{ 0.469305 f_1 + 0.5000000 f_2 \right\} = \frac{1}{3(1-\nu^2)} \left(\frac{t}{R}\right)^2 m^4 \left\{ 0.469305 f_1 + 0.5000000 f_2 \right\}$$

$$+ m^4 \left\{ 0.007267696 f_1^3 + 0.043385550 f_1^2 f_2 + 0.021093750 f_1 f_2^2 + 0.007812500 f_2^3 \right\}$$

$$- m^2 \left\{ 0.11493490 f_1^2 + 0.12057300 f_1 f_2 \right\} + \left\{ 0.11732625 f_1 + 0.1250000 f_2 \right\}$$

Substituting $g = f_1 + f_2$, $f_1 = g - f_2$

$$\frac{\sigma}{E} m^2 \left\{ 1.3040116 g - 0.83470661 f_2 \right\} = \frac{1}{3(1-\nu^2)} \left(\frac{t}{R}\right)^2 m^4 \left\{ 3.4612822 g - 2.9919777 f_2 \right\}$$

$$+ m^4 \left\{ 0.039596224 g^3 - 0.096985584 g^2 f_2 + 0.118567976 g f_2^2 - 0.054147386 f_2^3 \right\}$$

$$- m^2 \left\{ 0.13389054 g^2 - 0.03791128 g f_2 - 0.03569276 f_2^2 \right\} + \left\{ 0.23717658 g - 0.11965033 f_2 \right\}$$

$$\frac{\sigma}{E} m^2 \left\{ 0.469305 g + 0.030695 f_2 \right\} = \frac{1}{3(1-\nu^2)} \left(\frac{t}{R}\right)^2 m^4 \left\{ 0.469305 g + 0.030695 f_2 \right\}$$

$$+ m^4 \left\{ 0.007267696 g^3 + 0.02158240 g^2 f_2 - 0.04387415 g f_2^2 + 0.02283655 f_2^3 \right\}$$

$$- m^2 \left\{ 0.11493490 g^2 - 0.10929680 g f_2 - 0.00563810 f_2^2 \right\} + \left\{ 0.11732625 g + 0.00767375 f_2 \right\}$$

$$\frac{\sigma R}{Et}\gamma(\rho - 1.562239) = \frac{1}{2.73}\gamma^2(3.584465\rho - 4.146705) + (\gamma\xi)^2(0.0648700\rho^3 - 0.1420475\rho^2 + 0.1161912\rho - 0.0474373)$$

$$-(\gamma\xi)(0.0427608\rho^2 + 0.0454187\rho - 0.1604043) + (0.1435838\rho - 0.2841436)$$

$$\frac{\sigma R}{Et}\gamma(\rho + 15.28930) = \frac{1}{2.73}\gamma^2(\rho + 15.28930) + (\gamma\xi)^2(0.7439827\rho^3 - 1.4293582\rho^2 + 0.7031243\rho + 0.2367713)$$

$$-(\gamma\xi)(-0.1836814\rho^2 - 3.5607363\rho + 3.7444177) + (0.2500000\rho + 3.8223245)$$

$$\rho^2 + 15.28930\rho$$
$$- 1.562239\rho - 23.885541$$
$$-3.584465\rho^2 + 4.146705\rho$$
$$- 54.8039607\rho + 63.4002168$$

$$-2.584465\rho^2 - 36.93019\rho + 39.51468$$

$$(-0.9466905\rho^2 - 13.52754\rho + 14.47424)\gamma^2$$

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$$0.7437827\,\rho^4 - 1.4293582\,\rho^3 + 0.7031243\,\rho^2 + 0.2367713\,\rho$$
$$-1.1622788\,\rho^3 + 2.2329991\,\rho^2 - 1.0985859\,\rho - 0.3698934$$
$$-0.0641700\,\rho^4 + 0.1420475\,\rho^3 - 0.1161912\,\rho^2 + 0.0474373\,\rho$$
$$-0.9918169\,\rho^3 + 2.1718068\,\rho^2 - 1.7764821\,\rho + 0.7252831$$

$$(0.679113\,\rho^4 - 3.441406\,\rho^3 + 4.991739\,\rho^2 - 2.590859\,\rho + 0.355390)\,(f\xi)^2$$

$$-0.1836814\,\rho^3 - 3.5607363\,\rho^2 + 3.7444177\,\rho$$
$$+0.2469542\,\rho^2 + 5.5627211\,\rho - 5.84967536$$
$$-0.0427608\,\rho^3 - 0.0454187\,\rho^2 + 0.1604043\,\rho$$
$$-0.6537827\,\rho^2 - 0.6744201\,\rho + 2.4514675$$

$$(+0.226442\,\rho^3 + 3.972984\,\rho^2 - 8.773123\,\rho + 3.397206)\,(f\xi)$$

$$0.2500000\,\rho^2 + 3.8223245\,\rho$$
$$-0.3905598\,\rho - 5.9713844$$
$$-0.1435838\,\rho^2 + 0.2841436\,\rho$$
$$-2.1952958\,\rho + 4.344357$$

$$(0.106416\,\rho^2 + 1.520613\,\rho - 1.627027)$$

$$\left\{0.679113(\gamma\xi)^2\right\}\varrho^4 - \left\{3.441406(\gamma\xi)^2 - 0.226442(\gamma\xi)\right\}\varrho^3 + \left\{4.991739(\gamma\xi)^2 + 3.9729984(\gamma\xi) + 0.106416 - 0.946691\gamma^2\right\}\varrho^2$$

$$- \left\{2.590159(\gamma\xi)^2 + 8.773123(\gamma\xi) - 1.520613 + 13.52754\gamma^2\right\}\varrho +$$

$$+ \left\{0.355390(\gamma\xi)^2 + 3.397206(\gamma\xi) - 1.627027 + 14.47424\gamma^2\right\} = 0$$

$$\frac{\sigma R}{Et} = \frac{\gamma(1.3129991\varrho - 1.518940)}{(\varrho - 1.562239)} +$$

$$+ \frac{1}{\gamma(\varrho - 1.562239)}\Big[\left\{0.0648700(\gamma\xi)^2\right\}\varrho^3 - \left\{0.1420425(\gamma\xi)^2 + 0.0427608(\gamma\xi)\right\}\varrho^2$$

$$+ \left\{0.1161912(\gamma\xi)^2 - 0.0454187(\gamma\xi) + 0.1435838\right\}\varrho$$

$$- \left\{0.0474373(\gamma\xi)^2 - 0.1604043(\gamma\xi) + 0.2841436\right\}\Big]$$

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$\gamma = 0.100, \quad \xi = 20,$ $(\gamma\xi) = 2.0$

$(\gamma\xi)^2 = 4.0$

$\gamma^2 = 0.01000$

$2.716459\, \rho^4 - 13.312740\, \rho^3 + 28.009873\, \rho^2 - 26.524344\, \rho + 6.733687 = 0$

$F(\rho) = \rho^4 - 4.900782\, \rho^3 + 10.311196\, \rho^2 - 9.764332\, \rho + 2.478853 = 0$

$F'(\rho) = 4\rho^3 - 14.702346\, \rho^2 + 20.622392\, \rho - 9.764332$

$F(0.38 \quad) = 0.009279$ $\qquad F'(0.38 \quad) = -3.831$

002422

$F(0.382422) = 0.000033$ $\qquad F'(0.382422) = -3.804$

0000087

$F(0.382431) =$ O.K. $\qquad \rho_1 = 0.382431$

$F(\rho) = \rho^3 - 4.518351\, \rho^2 + 8.583239\, \rho - 6.481835 = 0$

$F'(\rho) = 3\rho^3 - 9.036702\, \rho + 8.583239$

$F(1.7 \quad) = 0.035364$ $\qquad F'(1.7) = 1.891$

.01868

$F(1.71858) = 0.000023$ $\qquad F'(1.71858) = 1.913$

000012

$F(1.718592) =$ O.K.

$\rho^2 - 2.799759\, \rho + 3.771596 = 0$ No more real root !!!

$\rho_2 = 1.718592$

$$\frac{OR}{Et} = \frac{0.100 \times 1.016812}{1.179808} - \frac{1}{0.1179808}\left\{0.25948\rho^3 - 0.6537116\rho^2 + 0.5175112\rho - 0.1530842\right\}$$

$$= 0.08618 + 0.30739 = \underline{\underline{0.39357}}$$

$\gamma = 0.100, \quad \xi = 30,$ $\quad (\gamma\xi) = 3.0$

$(\gamma\xi) = 9.0$

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$6.112017\,\rho^4 - 30.293328\,\rho^3 + 56.941552\,\rho^2 - 48.251762\,\rho + 11.907843 = 0$

$F(\rho) = \rho^4 - 4.956355\,\rho^3 + 9.311327\,\rho^2 - 7.894572\,\rho + 1.948267$

$F'(\rho) = 4\rho^3 - 14.869065\,\rho^2 + 18.632654\,\rho - 7.894572$

$F(0.37 \quad) =$

If $f_2 = 0$

$$\frac{\sigma R}{Et} = \gamma\left\{0.972283 + 0.030365\left(\frac{\ell}{t}\right)^2\right\} - 0.102675\left(\frac{\ell}{t}\right) + \frac{1}{\gamma}\,0.181882$$

$$= \sqrt{0.707363 + 0.022091\left(\frac{\ell}{t}\right)^2} - 0.102675\left(\frac{\ell}{t}\right)$$

$$0.022091\left(\frac{\ell}{t}\right) = 0.102675\left\{0.707363 + 0.022091\left(\frac{\ell}{t}\right)^2\right\}^{\frac{1}{2}}$$

$$\left(\frac{\ell}{t}\right)^2 = \frac{0.0105422 \times 0.707363}{0.022091\{0.022091 - 0.0105422\}} = \frac{0.0105422 \times 0.707363}{0.022091 \times 0.011549}$$

$$= 29.2290 \qquad \left(\frac{\ell}{t}\right) = 5.4064$$

$$\left(\frac{\sigma R}{Et}\right)_{min} = 5.4064\left\{\frac{0.022091}{0.102675} - 0.102675\right\}$$

$$= 0.5$$

$$\frac{w}{R} = f_0 + \frac{1}{2} f_1 \cos\frac{mx}{R}\cos\frac{my}{R} + f_2 \cos\frac{2mx}{R} + f_3 \cos\frac{2my}{R}$$

$$\frac{\partial w}{\partial x} = -m\left\{\frac{1}{2} f_1 \sin\frac{mx}{R}\cos\frac{my}{R} + 2 f_2 \sin\frac{2mx}{R}\right\}$$

$$\frac{\partial w}{\partial y} = -m\left\{\frac{1}{2} f_1 \cos\frac{mx}{R}\sin\frac{my}{R} + 2 f_3 \sin\frac{2my}{R}\right\}$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{m^2}{R}\left\{\frac{1}{2} f_1 \cos\frac{mx}{R}\cos\frac{my}{R} + 4 f_2 \cos\frac{2mx}{R}\right\}$$

$$\frac{\partial^2 w}{\partial y^2} = -\frac{m^2}{R}\left\{\frac{1}{2} f_1 \cos\frac{mx}{R}\cos\frac{my}{R} + 4 f_3 \cos\frac{2my}{R}\right\}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{m^2}{R}\left\{\frac{1}{2} f_1 \sin\frac{mx}{R}\sin\frac{my}{R}\right\}$$

Thus

$$\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2}$$

$$= \frac{m^4}{R^2}\Big[-\frac{1}{8} f_1^2 \cos\frac{2mx}{R} - \frac{1}{8} f_1^2 \cos\frac{2my}{R} - (f_1 f_2 + f_1 f_3)\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$- f_1 f_2 \cos\frac{3mx}{R}\cos\frac{my}{R} - f_1 f_3 \cos\frac{mx}{R}\cos\frac{3my}{R} - 16 f_2 f_3 \cos^2\frac{mx}{R}\cos\frac{2my}{R}\Big]$$

$$+ \frac{m^2}{R^2}\left[\frac{1}{2} f_1 \cos\frac{mx}{R}\cos\frac{my}{R} + 4 f_2 \cos\frac{2mx}{R}\right]$$

$$= -\frac{m^2}{R^2}\Big[\left\{\frac{1}{8} f_1^2 m^2 - 4 f_2\right\}\cos\frac{2mx}{R} + \frac{1}{8} f_1^2 m^2 \cos\frac{2my}{R} + \left\{(f_1 f_2 + f_1 f_3)m^2 - \frac{1}{2} f_1\right\}\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$+ f_1 f_2 m^2 \cos\frac{3mx}{R}\cos\frac{my}{R} + f_1 f_3 m^2 \cos\frac{mx}{R}\cos\frac{3my}{R} + 16 f_2 f_3 m^2 \cos\frac{2mx}{R}\cos\frac{2my}{R}\Big]$$

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$$\mathfrak{P}_1 = \frac{1}{8}\left\{\frac{1}{8} f_1^2 m^2 - k f_2\right\}^2 + \frac{1}{512}(f_1^2 m^2)^2 + \frac{1}{4}\left\{(f_1 f_2 + f_1 f_3) m^2 - \frac{1}{2} f_1\right\}^2$$

$$+ \frac{1}{100} f_1^2 f_2^2 m^4 + \frac{1}{100} f_1^2 f_3^2 m^4 + 4 f_2^2 f_3^2 m^4$$

$$= m^4 \left\{ \frac{1}{256} f_1^4 + \frac{1}{4} f_1^2 f_2^2 + \frac{1}{2} f_1^2 f_2 f_3 + \frac{1}{4} f_1^2 f_3^2 + \frac{1}{100} f_1^2 f_2^2 + \frac{1}{100} f_1^2 f_3^2 + 4 f_2^2 f_3^2 \right\}$$

$$- m^2 \left\{ \frac{1}{8} f_1^2 f_2 + \frac{1}{4} f_1^2 f_2 + \frac{1}{4} f_1^2 f_3 \right\} + \left\{ 2 f_2^2 + \frac{1}{16} f_1^2 \right\}$$

$$\mathfrak{P}_1 = m^4 \left\{ \frac{f_1^4}{256} + \frac{13}{50} f_1^2 f_2^2 + \frac{1}{2} f_1^2 f_2 f_3 + \frac{13}{50} f_1^2 f_3^2 + 4 f_2^2 f_3^2 \right\}$$

$$- m^2 \left\{ \frac{3}{8} f_1^2 f_2 + \frac{1}{4} f_1^2 f_3 \right\} + \left\{ \frac{1}{16} f_1^2 + 2 f_2^2 \right\}$$

$$K = -4\left(\frac{\sigma}{E}\right)^2 - m^2 \frac{\sigma}{E}\left[\frac{1}{4} f_1^2 + 8 f_2^2\right]$$

$$\mathfrak{P}_2 = \frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \left[f_1^2 + 32 f_2^2 + 32 f_3^2 \right]$$

$$\frac{1}{2} f_1 \frac{\sigma}{E} m^2 = m^4\left\{\frac{1}{64} f_1^3 + \frac{13}{25} f_1 f_2^2 + f_1 f_2 f_3 + \frac{13}{25} f_1 f_3^2\right\} - m^2\left\{\frac{3}{4} f_1 f_2 + \frac{1}{2} f_1 f_3\right\} + \left\{\frac{1}{8} f_1\right\}$$
$$+ \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 f_1$$

$$16 f_2 \frac{\sigma}{E} m^2 = m^4\left\{\frac{13}{25} f_1^2 f_2 + \frac{1}{2} f_1^2 f_3 + 8 f_2 f_3^2\right\} - m^2\left\{\frac{3}{8} f_1^2\right\} + \left\{4 f_2\right\} + \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4\, 32 f_2$$

$$0 = m^4\left\{\frac{1}{2} f_1^2 f_2 + \frac{13}{25} f_1^2 f_3 + 8 f_2^2 f_3\right\} - m^2\left\{\frac{1}{4} f_1^2\right\} + 0 + \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4\, 32 f_3$$

$$\frac{\sigma R}{Et}\gamma = (\gamma\xi)^2\left\{\frac{1}{32} + \frac{26}{25}\alpha^2 + 2\alpha\beta + \frac{26}{25}\beta^2\right\} - (\gamma\xi)\left\{\frac{3}{2}\alpha + \beta\right\} + \frac{1}{4} + \frac{1}{3(1-\nu^2)}\gamma^2 \qquad (1)$$

$$\frac{\sigma R}{Et}\gamma\alpha = (\gamma\xi)^2\left\{\frac{13}{400}\alpha + \frac{1}{32}\beta + \frac{1}{2}\alpha\beta^2\right\} - (\gamma\xi)\frac{3}{128} + \frac{1}{4}\alpha + \frac{1}{3(1-\nu^2)}\gamma^2\alpha \qquad (2)$$

$$0 = (\gamma\xi)^2\left\{\frac{1}{32}\alpha + \frac{13}{400}\beta + \frac{1}{2}\alpha^2\beta\right\} - (\gamma\xi)\frac{1}{64} + 0 + \frac{1}{3(1-\nu^2)}\gamma^2\beta \qquad (3)$$

$$0 = (\gamma\xi)\left\{\left(\frac{1}{32} - \frac{13}{400}\right)\alpha + \frac{26}{25}\alpha^3 + 2\alpha^2\beta + \left(\frac{26}{25} - \frac{1}{2}\right)\alpha\beta^2 - \frac{1}{32}\beta\right\}$$
$$- \left\{\frac{3}{2}\alpha^2 + \alpha\beta - \frac{3}{128}\right\}$$

$$(\gamma\xi)\left\{-\frac{1}{800}\alpha+\frac{26}{25}\alpha^3+2\alpha^2\beta+\frac{27}{50}\alpha\beta^2-\frac{1}{32}\beta\right\}-\left\{\frac{3}{2}\alpha^2+\alpha\beta-\frac{3}{128}\right\}=0 \qquad (4)$$

But from (3),

$$\left\{\left(\frac{1}{2}\alpha^2+\frac{13}{400}\right)(\gamma\xi)^2+\frac{1}{3(1-\nu^2)}\gamma^2\right\}\beta+\frac{1}{32}\left\{(\gamma\xi)^2\alpha-\frac{1}{2}(\gamma\xi)\right\}=0$$

$$\therefore \boxed{\beta=(\gamma\xi)\left\{\frac{\frac{1}{2}-(\gamma\xi)\alpha}{\left(16\alpha^2+\frac{26}{25}\right)(\gamma\xi)^2+\frac{32}{3(1-\nu^2)}\gamma^2}\right\}}$$

$$\left\{\left(\frac{26}{25}\alpha^3-\frac{1}{800}\alpha\right)(\gamma\xi)-\frac{3}{2}\alpha^2+\frac{3}{128}\right\}\left\{\left(16\alpha^2+\frac{26}{25}\right)(\gamma\xi)^2+\frac{32}{3(1-\nu^2)}\gamma^2\right\}^2$$

$$+\left\{\left(2\alpha^2-\frac{1}{32}\right)(\gamma\xi)-\alpha\right\}\left\{\left(16\alpha^2+\frac{26}{25}\right)(\gamma\xi)^2+\frac{32}{3(1-\nu^2)}\gamma^2\right\}\left\{\frac{1}{2}\gamma\xi-\alpha(\gamma\xi)^2\right\}$$

$$+\frac{27}{50}\alpha(\gamma\xi)^3\left\{\frac{1}{2}-(\gamma\xi)\alpha\right\}^2=0$$

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$$\left\{\frac{26}{25}(\gamma\xi)\,\alpha^3-\frac{3}{2}\alpha^2-\frac{1}{800}(\gamma\xi)\,\alpha+\frac{3}{128}\right\}\left\{256(\gamma\xi)^4\alpha^4+2(\gamma\xi)^2\left[\frac{26}{25}(\gamma\xi)^2+\frac{32}{3(1-\nu^2)}\gamma^2\right]16\alpha^2+\left[\frac{26}{25}(\gamma\xi)^2+\frac{32}{3(1-\nu^2)}\gamma^2\right]^2\right\}$$

$$-\left\{2(\gamma\xi)^3\alpha^3-2(\gamma\xi)^2\alpha^2+\left[\frac{1}{2}(\gamma\xi)-\frac{1}{32}(\gamma\xi)^3\right]\alpha+\frac{1}{64}(\gamma\xi)^2\right\}\left\{16(\gamma\xi)^2\alpha^2+\left[\frac{26}{25}(\gamma\xi)^2+\frac{32}{3(1-\nu^2)}\gamma^2\right]\right\}$$

$$+\frac{27}{50}(\gamma\xi)^3\,\alpha\left\{(\gamma\xi)^2\alpha^2-(\gamma\xi)\alpha+\frac{1}{4}\right\}=0$$

α^7	α^6	α^5	α^4	α^3	α^2	α	1
$\frac{26}{25}(\gamma\xi)^5 256$	$-\frac{3}{2}(\gamma\xi)^4\cdot 256$	$\frac{52}{25}(\gamma\xi)^3[\quad]16$ $-\frac{1}{800}(\gamma\xi)^5 256$	$-3(\gamma\xi)^2[\quad]16$ $\frac{3}{128}(\gamma\xi)^4 256$	$\frac{26}{25}(\gamma\xi)[\quad]^2$ $-\frac{1}{400}(\gamma\xi)^3[\quad]16$	$-\frac{3}{2}[\quad]^2$ $\frac{3}{64}(\gamma\xi)^2[\quad]16$	$-\frac{1}{800}(\gamma\xi)[\quad]^2$	$\frac{3}{128}[\quad]^2$
		$-2(\gamma\xi)^5 16$	$+2(\gamma\xi)^4 16$	$-\frac{1}{2}(\gamma\xi)^3[1-\frac{1}{16}(\gamma\xi)^2]16$ $-2(\gamma\xi)^3[\quad]$	$-\frac{1}{64}(\gamma\xi)^4 16$ $+2(\gamma\xi)^2[\quad]$	$-\frac{1}{2}(\gamma\xi)[1-\frac{1}{16}(\gamma\xi)^2][\quad]$	$-\frac{1}{64}(\gamma\xi)^2[\quad]$
				$\frac{27}{50}(\gamma\xi)^5$	$-\frac{27}{50}(\gamma\xi)^4$	$\frac{27}{200}(\gamma\xi)^3$	

$$A_7\alpha^7 + A_6\alpha^6 + A_5\alpha^5 + A_4\alpha^4 + A_3\alpha^3 + A_2\alpha^2 + A_1\alpha + A_0 = 0$$

$$\boxed{A_7 = +\ 266.24(\gamma\xi)^5}$$

$$\boxed{A_6 = -\ 384(\gamma\xi)^4}$$

$$A_5 = (\gamma\xi)^3\left\{-\frac{808}{25}(\gamma\xi)^2 + \frac{21632}{25^2}(\gamma\xi)^2 + \frac{26624}{75(1-\nu^2)}\gamma^2\right\} = (\gamma\xi)^3\left\{\frac{1432}{625}(\gamma\xi)^2 + \frac{26624}{75(1-\nu^2)}\gamma^2\right\}$$

$$\boxed{A_5 = (\gamma\xi)^3\left\{2.29120(\gamma\xi)^2 + 390.09524\gamma^2\right\}}$$

$$A_4 = (\gamma\xi)^2\left\{38(\gamma\xi)^2 - \frac{1248}{25}(\gamma\xi)^2 - \frac{512}{(1-\nu^2)}\gamma^2\right\} = -(\gamma\xi)^2\left\{\frac{298}{25}(\gamma\xi)^2 + \frac{512}{(1-\nu^2)}\gamma^2\right\}$$

$$\boxed{A_4 = -(\gamma\xi)^2\left\{11.92000(\gamma\xi)^2 + 562.63736\gamma^2\right\}}$$

$$A_3 = (\gamma\xi)^3\left\{\frac{27}{50}(\gamma\xi)^2 - 8 + \frac{1}{2}(\gamma\xi)^2\right\} + \left\{\frac{26}{25}(\gamma\xi)^2 + \frac{32}{3(1-\nu^2)}\gamma^2\right\}\left\{-\left(2+\frac{1}{25}\right)(\gamma\xi)^2 + \frac{676}{625}(\gamma\xi)^2 + \frac{832}{75(1-\nu^2)}\gamma^2\right\}(\gamma\xi)$$

$$= (\gamma\xi)^3\left\{\frac{26}{25}(\gamma\xi)^2 - 8\right\} - (\gamma\xi)\left\{\frac{26}{25}(\gamma\xi)^2 + \frac{32}{3(1-\nu^2)}\gamma^2\right\}\left\{\frac{599}{625}(\gamma\xi)^2 - \frac{832}{75(1-\nu^2)}\gamma^2\right\}$$

$$A_3 = -(\gamma\xi)^3\left\{8 - 1.04000(\gamma\xi)^2\right\} - (\gamma\xi)\left\{1.04000(\gamma\xi)^2 + 11.7216117\gamma^2\right\}\left\{0.958400(\gamma\xi)^2 - 12.190476\gamma^2\right\}$$

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$$A_3 = -(\gamma\xi)\left\{-0.04326400(\gamma\xi)^4 + (8 - 1.4441024\gamma^2)(\gamma\xi)^2 - 142.892030\gamma^4\right\}$$

$$A_2 = -\frac{79}{100}(\gamma\xi)^4 + \left\{\frac{26}{25}(\gamma\xi)^2 + \frac{32}{3(1-\nu^2)}\gamma^2\right\}\left\{\frac{11}{4}(\gamma\xi)^2 - \frac{78}{50}(\gamma\xi)^2 - \frac{96}{6(1-\nu^2)}\gamma^2\right\}$$

$$= -0.79(\gamma\xi)^4 + \left\{1.04(\gamma\xi)^2 + 11.7216117\gamma^2\right\}\left\{1.19000(\gamma\xi)^2 - 17.5824176\gamma^2\right\}$$

$$A_2 = \left\{0.4476000(\gamma\xi)^4 - 4.3369964\gamma^2(\gamma\xi)^2 - 206.094272\gamma^4\right\}$$

$$A_1 = \frac{27}{200}(\gamma\xi)^3 - (\gamma\xi)\left\{0.52(\gamma\xi)^2 + 5.8608059\gamma^2\right\}\left\{1 - 0.0597(\gamma\xi)^2 + 0.029304029\gamma^2\right\}$$

$$= -(\gamma\xi)\left\{0.385(\gamma\xi)^2 + 5.8608059\gamma^2\right\} + (\gamma\xi)\left\{0.52(\gamma\xi)^2 + 5.8608059\gamma^2\right\}\left\{0.0599(\gamma\xi)^2 - 0.029304029\gamma^2\right\}$$

$$A_1 = -(\gamma\xi)\left\{-0.031148(\gamma\xi)^4 + (0.385 - 0.3358242\gamma^2)(\gamma\xi)^2 + (0.1717452\gamma^2 + 5.8608059)\gamma^2\right\}$$

$$A_0 = 0.0625\left\{0.145600(\gamma\xi)^4 + 6.2124542\gamma^2(\gamma\xi)^2 + 51.523568\gamma^4\right\}$$

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$\gamma = 0.1000, \quad \xi = 16$

$(\gamma\xi) = 1.6$, $(\gamma\xi)^2 = 2.56$, $(\gamma\xi)^3 = 4.096$, $(\gamma\xi)^4 = 6.5536$, $(\gamma\xi)^5 = 10.48576$

$A_7 = 2791.7287$ | $7A_7 = 19542.1009$
$A_6 = -2516.5824$ | $6A_6 = -15099.4944$
$A_5 = +40.0032743$ | $5A_5 = +200.0163715$
$A_4 = -92.5224284$ | $4A_4 = -370.0897136$
$A_3 = -32.2313666$ | $3A_3 = -96.6940998$
$A_2 = +2.8017548$ | $2A_2 = +5.6035096$
$A_1 = -0.8314967$ | $A_1 = -0.8314967$
$A_0 = +0.06989971$

$F(0.08\ \ ) = +0.00054895$ $F'(0.08) = 1.2277$
$\quad 0004471$

$F(0.0804471) = -0.00000189$ $F'(0.0804471) = 1.236$
$\quad 0000015$

$F(0.0804456) =$ O.K.

$\alpha_1 = +0.0804456$

$A_6' = 2791.7287$ $6A_6' = 16750.3722$ 731

$A_5' = -2292.0001$ $5A_5' = -11460.0005$

$A_4' = -144.37805$ $4A_4' = -577.51220$

$A_3' = -104.137007$ $3A_3' = -312.411021$

$A_2' = -40.608731$ $2A_2' = -81.217462$

$A_1' = -0.4650389$ $A_1' = -0.4650389$

$A_0' = -0.8689070$

$F(0.9370\,0000048) = -0.011715$ $F'(0.9370) = 2439$

$\alpha_2 = 0.9370048$

$$\beta_2 = -\frac{1.60000 \times 0.9992077}{38.7415950} = -0.0412666$$

$$\frac{\sigma R}{Et} = 25.6\{0.03125 + 1.04\alpha^2 + 2\alpha\beta + 1.04\beta^2\} - 16\{1.5\alpha + \beta\} + 2.5$$

$+ 0.036630$

$=$ ///

$(-\alpha)$

$$F(\alpha) = 2791.7287\,\alpha^5 - 323.86309\,\alpha^4 + 159.083222\,\alpha^3 - 44.924236\,\alpha^2 + 1.485962\,\alpha - 0.927315 = 0$$

$$F'(\alpha) = 13958.6435\,\alpha^4 - 1295.45236\,\alpha^3 + 477.249666\,\alpha^2 - 89.85472\,\alpha + 1.485962$$

$F(0.235) = +\ 0.018603$ $\qquad F'(0.235) = 32.485$

0005726

$F(0.2344274) = +0.000106$ $\qquad F'(0.2344274) = 32.117$

0000033

$F(0.2344241) =$ o.K.

$$\alpha_3 = -\ 0.2344241$$

$$\beta_3 = \frac{1.6000 \times 0.87507856}{5.0305588} = +0.278324101$$

$$\left(\frac{\sigma R}{Et}\right)_3 = 25.6\left\{0.03125 + 0.0571529 - 0.1304918 + 0.0805629\right\} + 16 \times 0.0733121 + 2.5 + 0.0366300$$

$$= \underline{\underline{4.6946}}\ !!!$$

$$\left(\frac{\sigma R}{Et}\right)_3 = 25.6\left\{\frac{13}{400} + \frac{1}{32}\frac{\beta}{\alpha} + \frac{1}{2}\beta^2\right\} - 16\,\frac{3}{128}\,\frac{1}{\alpha} + 2.5 + 0.0366300$$

$$= 25.6\left\{0.0325 - 0.0371021 + 0.0387322\right\} + 1.5996648 + 2.5 + 0.0366300$$

$$= \underline{\underline{5.010036}}\ !!!$$

Contd to p. 677 !!!

$$\frac{w}{R} = (f_0 + \tfrac{1}{4}f_1) + \tfrac{1}{2}f_1\left[\cos\frac{mx}{R}\cos\frac{ny}{R} + \tfrac{1}{4}\cos\frac{2mx}{R} + \tfrac{1}{4}\cos\frac{2ny}{R}\right]$$
$$+ \tfrac{1}{4}f_2\left[\cos\frac{2mx}{R} + \cos\frac{2ny}{R}\right]$$

$$\frac{w}{R} = (f_0 + \tfrac{1}{4}f_1) + \tfrac{1}{2}f_1\cos\frac{mx}{R}\cos\frac{ny}{R} + \tfrac{1}{4}(\tfrac{1}{2}f_1 + f_2)\cos\frac{2mx}{R} + \tfrac{1}{4}(\tfrac{1}{2}f_1 + f_2)\cos\frac{2ny}{R}$$

$$\frac{\partial w}{\partial x} = -n\left[\tfrac{1}{2}\left(\frac{m}{n}\right)f_1\sin\frac{mx}{R}\cos\frac{ny}{R} + \tfrac{1}{2}\left(\frac{m}{n}\right)(\tfrac{1}{2}f_1 + f_2)\sin\frac{2mx}{R}\right]$$
$$\frac{\partial w}{\partial y} = -n\left[\tfrac{1}{2}f_1\cos\frac{mx}{R}\sin\frac{ny}{R} + \tfrac{1}{2}(\tfrac{1}{2}f_1 + f_2)\sin\frac{2ny}{R}\right]$$
$$\frac{1}{R}\frac{\partial^2 w}{\partial x^2} = -\left(\frac{n}{R}\right)^2\left[\tfrac{1}{2}\left(\frac{m}{n}\right)^2 f_1\cos\frac{mx}{R}\cos\frac{ny}{R} + \left(\frac{m}{n}\right)^2(\tfrac{1}{2}f_1 + f_2)\cos\frac{2mx}{R}\right]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial y^2} = -\left(\frac{n}{R}\right)^2\left[\tfrac{1}{2}f_1\cos\frac{mx}{R}\cos\frac{ny}{R} + (\tfrac{1}{2}f_1 + f_2)\cos\frac{2ny}{R}\right]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x\partial y} = +\left(\frac{n}{R}\right)^2\left[\tfrac{1}{2}\left(\frac{m}{n}\right)\sin\frac{mx}{R}\sin\frac{ny}{R}\right] \qquad \left(\mu = \frac{m}{n}\right)$$

$$\Delta\Delta F = E\left(\frac{n}{R}\right)^2\Big[n^2\Big\{-\tfrac{1}{8}\mu^2 f_1^2\left(\cos\frac{2mx}{R} + \cos\frac{2ny}{R}\right) - \tfrac{1}{4}\mu^2 f_1(\tfrac{1}{2}f_1 + f_2)\left(\cos\frac{mx}{R} + \cos\frac{3mx}{R}\right)\cos\frac{ny}{R}$$
$$- \tfrac{1}{4}\mu^2 f_1(\tfrac{1}{2}f_1 + f_2)\cos\frac{mx}{R}\left(\cos\frac{ny}{R} + \cos\frac{3ny}{R}\right)$$
$$- \mu^2(\tfrac{1}{2}f_1 + f_2)^2\cos\frac{2mx}{R}\cos\frac{2ny}{R} + \tfrac{1}{2}\mu^2 f_1\cos\frac{mx}{R}\cos\frac{ny}{R} + \mu^2(\tfrac{1}{2}f_1 + f_2)\cos\frac{2mx}{R}\Big]$$
$$= -E\mu^2\left(\frac{n}{R}\right)^2\Big[\left\{\tfrac{1}{8}f_1^2 n^2 - (\tfrac{1}{2}f_1 + f_2)\right\}\cos\frac{2mx}{R} + \tfrac{1}{4}f_1(\tfrac{1}{2}f_1 + f_2)n^2\cos\frac{3mx}{R}\cos\frac{ny}{R}$$
$$+ \tfrac{1}{4}f_1(\tfrac{1}{2}f_1 + f_2)n^2\cos\frac{mx}{R}\cos\frac{3ny}{R} + \left\{\tfrac{1}{2}f_1(\tfrac{1}{2}f_1 + f_2)n^2 - \tfrac{1}{2}f_1\right\}\cos\frac{mx}{R}\cos\frac{ny}{R}$$
$$+ (\tfrac{1}{2}f_1 + f_2)^2 n^2\cos\frac{2mx}{R}\cos\frac{2ny}{R} + \tfrac{1}{8}f_1^2 n^2\cos\frac{2ny}{R}\Big]$$

$$F = -E\mu^2\left(\frac{R}{n}\right)^2\Big[\frac{1}{(2\mu)^4}\Big\{\frac{1}{8}f_1^2n^2-\left(\frac{1}{2}f_1+f_2\right)\Big\}\cos\frac{2mx}{R}+\frac{1}{2^4}\frac{1}{8}f_1^2n^2\cos\frac{2ny}{R}$$

$$+\frac{1}{(\mu^2+1)^2}\Big\{\frac{1}{2}f_1\left(\frac{1}{2}f_1+f_2\right)n^2-\frac{1}{2}f_1\Big\}\cos\frac{mx}{R}\cos\frac{ny}{R}+\frac{1}{4}\frac{1}{(9\mu^2+1)^2}f_1\left(\frac{1}{2}f_1+f_2\right)n^2\cos\frac{3mx}{R}\cos\frac{ny}{R}$$

$$+\frac{1}{4}\frac{1}{(\mu^2+9)^2}f_1\left(\frac{1}{2}f_1+f_2\right)n^2\cos\frac{mx}{R}\cos\frac{3ny}{R}+\frac{1}{16(\mu^2+1)^2}\left(\frac{1}{2}f_1+f_2\right)^2n^2\cos\frac{2mx}{R}\cos\frac{2ny}{R}\Big]$$

$$\sigma_x = E\mu^2\Big[\frac{1}{32}f_1^2n^2\cos\frac{2ny}{R}+\frac{1}{(\mu^2+1)^2}\Big\{\frac{1}{2}f_1\left(\frac{1}{2}f_1+f_2\right)n^2-\frac{1}{2}f_1\Big\}\cos\frac{mx}{R}\cos\frac{ny}{R}$$

$$+\frac{1}{4}\frac{1}{(9\mu^2+1)^2}f_1\left(\frac{1}{2}f_1+f_2\right)n^2\cos\frac{3mx}{R}\cos\frac{ny}{R}+\frac{1}{4}\frac{9}{(\mu^2+9)^2}f_1\left(\frac{1}{2}f_1+f_2\right)n^2\cos\frac{mx}{R}\cos\frac{3ny}{R}$$

$$+\frac{1}{4}\frac{1}{(\mu^2+1)^2}\left(\frac{1}{2}f_1+f_2\right)^2n^2\cos\frac{2mx}{R}\cos\frac{2ny}{R}\Big]$$

$$\sigma_y = E\mu^2\Big[\frac{1}{(2\mu)^2}\Big\{\frac{1}{8}f_1^2n^2-\left(\frac{1}{2}f_1+f_2\right)\Big\}\cos\frac{2mx}{R}+\frac{\mu^2}{(\mu^2+1)^2}\Big\{\frac{1}{2}f_1\left(\frac{1}{2}f_1+f_2\right)n^2-\frac{1}{2}f_1\Big\}\cos\frac{mx}{R}\cos\frac{ny}{R}$$

$$+\frac{1}{4}\frac{9\mu^2}{(9\mu^2+1)^2}f_1\left(\frac{1}{2}f_1+f_2\right)n^2\cos\frac{3mx}{R}\cos\frac{ny}{R}+\frac{1}{4}\frac{\mu^2}{(\mu^2+9)^2}f_1\left(\frac{1}{2}f_1+f_2\right)n^2\cos\frac{mx}{R}\cos\frac{3ny}{R}$$

$$+\frac{1}{4}\frac{\mu^2}{(\mu^2+1)^2}f_1\left(\frac{1}{2}f_1+f_2\right)^2n^2\cos\frac{2mx}{R}\cos\frac{2ny}{R}\Big]$$

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$$\wp_1 = \mu^4 \Big[\frac{2}{(2\mu)^4} \Big\{ \frac{1}{8} f_1^2 n^2 - \Big(\frac{1}{2} f_1 + f_2\Big) \Big\}^2 + \frac{1}{512} f_1^4 n^4 + \frac{1}{4(\mu^2+1)^2} \Big\{ f_1 \Big(\frac{1}{2} f_1 + f_2\Big) n^2 - f_1 \Big\}^2$$

$$+ \frac{1}{16} \frac{1}{(9\mu^2+1)^2} f_1^2 n^4 \Big(\frac{1}{2} f_1 + f_2\Big)^2 + \frac{1}{16} \frac{1}{(\mu^2+9)^2} f_1^2 n^4 \Big(\frac{1}{2} f_1 + f_2\Big)^2 + \frac{1}{16} \frac{1}{(\mu^2+1)^2} \Big(\frac{1}{2} f_1 + f_2\Big)^4 n^4 \Big]$$

$$= \frac{\mu^4}{4} \Big[\frac{1}{2\mu^4} \Big\{ \frac{1}{64} f_1^4 n^4 - \Big(\frac{1}{8} f_1^3 n^2 + \frac{1}{4} f_1^2 f_2 n^2 \Big) + \Big(\frac{1}{4} f_1^2 + f_1 f_2 + f_2^2 \Big) \Big\} + \frac{1}{128} f_1^4 n^4$$

$$+ \frac{1}{(\mu^2+1)^2} \Big\{ \Big(\frac{1}{4} f_1^4 n^4 + f_1^3 f_2 n^4 + f_1^2 f_2^2 n^4 \Big) - \Big(f_1^3 n^2 + 2 f_1^2 f_2 n^2 \Big) + f_1^2 \Big\}$$

$$+ \frac{1}{4} \Big\{ \frac{1}{(9\mu^2+1)^2} + \frac{1}{(\mu^2+9)^2} \Big\} \Big\{ \frac{1}{4} f_1^4 n^4 + f_1^3 f_2 n^4 + f_1^2 f_2^2 n^4 \Big\} + \frac{1}{4} \frac{1}{(\mu^2+1)^2} \Big\{ \frac{1}{16} f_1^4 n^4 + \frac{1}{2} f_1^3 f_2 n^4 + \frac{3}{2} f_1^2 f_2^2 n^4$$

$$+ 2 f_1 f_2^3 n^4 + f_2^4 n^4 \Big\} \Big]$$

$$= \frac{\mu^4}{4} \Big[n^4 \Big\{ f_1^4 \Big(\frac{1}{128\mu^4} + \frac{1}{128} + \frac{1}{4(\mu^2+1)^2} + \frac{1}{16(9\mu^2+1)^2} + \frac{1}{16(\mu^2+9)^2} + \frac{1}{64(\mu^2+1)^2} \Big)$$

$$+ f_1^3 f_2 \Big(\frac{1}{(\mu^2+1)^2} + \frac{1}{4(9\mu^2+1)^2} + \frac{1}{4(\mu^2+9)^2} + \frac{1}{8(\mu^2+1)^2} \Big)$$

$$+ f_2^2 f_2^2 \Big(\frac{1}{(\mu^2+1)^2} + \frac{1}{4(9\mu^2+1)^2} + \frac{1}{4(\mu^2+9)^2} + \frac{3}{8(\mu^2+1)^2} \Big) + \frac{1}{2(\mu^2+1)^2} f_1 f_2^3 + \frac{1}{4(\mu^2+1)^2} f_2^4 \Big\}$$

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$$-n^2\left\{ f_1^3\left(\frac{1}{16\mu^4}+\frac{1}{(\mu^2+1)^2}\right)+f_1^2f_2\left(\frac{1}{8\mu^4}+\frac{2}{(\mu^2+1)^2}\right)\right\}+\left\{ f_1^2\left(\frac{1}{8\mu^4}+\frac{1}{(\mu^2+1)^2}\right)\right.$$

$$\left.+\frac{1}{2\mu^4}f_1f_2+\frac{1}{2\mu^4}f_2^2\right\}$$

$$\delta_1=\frac{1}{4}\left\{ n^4\left[\left\{\frac{(1+\mu^4)}{128}+\frac{17}{64}\frac{\mu^4}{(\mu^2+1)^2}+\frac{\mu^4}{16(9\mu^2+1)^2}+\frac{\mu^4}{16(\mu^2+9)^2}\right\} f_1^4\right.\right.$$

$$+\left\{\frac{9}{8}\frac{\mu^4}{(\mu^2+1)^2}+\frac{\mu^4}{4(9\mu^2+1)^2}+\frac{\mu^4}{4(\mu^2+9)^2}\right\} f_1^3f_2+\left\{\frac{11}{8}\frac{\mu^4}{(\mu^2+1)^2}+\frac{\mu^4}{4(9\mu^2+1)^2}+\frac{\mu^4}{4(\mu^2+9)^2}\right\} f_1^2f_2^2$$

$$\left.+\frac{\mu^4}{2(\mu^2+1)^2}f_1f_2^3+\frac{\mu^4}{4(\mu^2+1)^2}f_2^4\right]$$

$$\left.-n^2\left[\left\{\frac{1}{16}+\frac{\mu^4}{(\mu^2+1)^2}\right\} f_1^3+\left\{\frac{1}{8}+\frac{2\mu^4}{(\mu^2+1)^2}\right\} f_1^2f_2\right]+\left[\left\{\frac{1}{8}+\frac{\mu^4}{(\mu^2+1)^2}\right\} f_1^2+\frac{1}{2}f_1f_2+\frac{1}{2}f_2^2\right]\right\}$$

$$\delta_2=\frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2n^4\left\{\frac{1}{4}(1+\mu^2)^2f_1^2+2\mu^4\left(\frac{1}{2}f_1+f_2\right)^2+2\left(\frac{1}{2}f_1+f_2\right)^2\right\}$$

$$\delta_2=\frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2n^4\left\{ f_1^2\left[\frac{1}{8}(1+\mu^2)^2+\frac{1}{4}(1+\mu^4)\right]+(1+\mu^4)f_1f_2+(1+\mu^4)f_2^2\right\}$$

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$$K = -4\left(\frac{\sigma}{E}\right)^2 - n^2 \frac{\sigma}{E}\mu^2\left[\frac{3}{8}f_1^2 + \frac{1}{2}f_1 f_2 + \frac{1}{2}f_2^2\right]$$

$$\frac{\sigma R}{Et}\gamma^2\mu^2\left(\lambda + \frac{3}{2}\right) = \left\{(\gamma\xi)^2\left[\frac{\mu^4}{4(\mu^2+1)^2}\lambda^3 + \left\{\frac{11}{8}\frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{4(9\mu^2+1)^2} + \frac{\mu^4}{4(\mu^2+9)^2}\right\}\lambda^2\right.\right.$$

$$+\left\{\frac{27}{16}\frac{\mu^4}{(\mu^2+1)^2} + \frac{3\mu^4}{8(9\mu^2+1)^2} + \frac{3\mu^4}{8(\mu^2+9)^2}\right\}\lambda + \left\{\frac{1+\mu^4}{64} + \frac{17}{32}\frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{8(9\mu^2+1)^2} + \frac{\mu^4}{8(\mu^2+9)^2}\right\}\Bigg]$$

$$-(\gamma\xi)\left[\left\{\frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2}\right\}\lambda + \left\{\frac{3}{32} + \frac{3\mu^4}{2(\mu^2+1)^2}\right\}\right] + \left[\frac{1}{4}\lambda + \left\{\frac{1}{8} + \frac{\mu^4}{(\mu^2+1)^2}\right\}\right]\Bigg\}$$

$$+\frac{1}{3(1-\nu^2)}\gamma^2\left\{(1+\mu^4)\lambda + \left[\frac{1}{4}(1+\mu^2)^2 + \frac{1}{2}(1+\mu^4)\right]\right\}$$

$$\frac{\sigma R}{Et}\gamma\mu^2\left(\lambda + \frac{1}{2}\right) = \left\{(\gamma\xi)^2\left[\frac{\mu^4}{4(\mu^2+1)^2}\lambda^3 + \frac{3\mu^4}{8(\mu^2+1)^2}\lambda^2 + \left\{\frac{11}{16}\frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{8(9\mu^2+1)^2} + \frac{\mu^4}{8(\mu^2+9)^2}\right\}\lambda\right.\right.$$

$$+\left\{\frac{9}{32}\frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{16(9\mu^2+1)^2} + \frac{\mu^4}{16(\mu^2+9)^2}\right\}\Bigg] - (\gamma\xi)\left[\left\{\frac{1}{32} + \frac{\mu^4}{2(\mu^2+1)^2}\right\}\right] + \left[\frac{\lambda}{4} + \frac{1}{8}\right]\Bigg\}$$

$$+\frac{1}{3(1-\nu^2)}\gamma^2\left\{2(1+\mu^4)\lambda + \frac{1}{2}(1+\mu^4)\right\}$$

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$$\frac{\mu^4}{4(\mu^2+1)^2}\lambda^3\left\{\lambda+\frac{1}{2}-\lambda-\frac{3}{2}\right\}=-\frac{\mu^4}{4(\mu^2+1)^2}\lambda^3$$

$$\left(\lambda+\frac{1}{2}\right)\left\{\frac{11}{8}\frac{\mu^4}{(\mu^2+1)^2}+\frac{\mu^4}{4(9\mu^2+1)^2}+\frac{\mu^4}{4(\mu^2+9)^2}\right\}\lambda^2-\left(\lambda+\frac{3}{2}\right)\frac{3\mu^4}{8(\mu^2+1)^2}\lambda^2$$

$$=\lambda^3\left\{\frac{\mu^4}{(\mu^2+1)^2}+\frac{\mu^4}{4(9\mu^2+1)^2}+\frac{\mu^4}{4(\mu^2+9)^2}\right\}+\lambda^2\left\{\frac{1}{8}\frac{\mu^4}{(\mu^2+1)^2}+\frac{\mu^4}{8(9\mu^2+1)^2}+\frac{\mu^4}{8(\mu^2+9)^2}\right\}$$

$$\left(\lambda+\frac{1}{2}\right)\left\{\frac{27}{16}\frac{\mu^4}{(\mu^2+1)^2}+\frac{3}{8}\frac{\mu^4}{(9\mu^2+1)^2}+\frac{3}{8}\frac{\mu^4}{(\mu^2+9)^2}\right\}\lambda-\left(\lambda+\frac{3}{2}\right)\left\{\frac{11}{16}\frac{\mu^4}{(\mu^2+1)^2}+\frac{\mu^4}{8(9\mu^2+1)^2}+\frac{\mu^4}{8(\mu^2+9)^2}\right\}\lambda$$

$$=\lambda^2\left\{\frac{\mu^4}{(\mu^2+1)^2}+\frac{1}{4}\frac{\mu^4}{(9\mu^2+1)^2}+\frac{1}{4}\frac{\mu^4}{(\mu^2+9)^2}\right\}+\lambda\left\{-\frac{3}{16}\frac{\mu^4}{(\mu^2+1)^2}+0+0\right\}$$

$$\left(\lambda+\frac{1}{2}\right)\left\{\frac{1+\mu^4}{64}+\frac{17}{32}\frac{\mu^4}{(\mu^2+1)^2}+\frac{\mu^4}{8(9\mu^2+1)^2}+\frac{\mu^4}{8(\mu^2+9)^2}\right\}-\left(\lambda+\frac{3}{2}\right)\left\{\frac{9}{32}\frac{\mu^4}{(\mu^2+1)^2}+\frac{\mu^4}{16(9\mu^2+1)^2}+\frac{\mu^4}{16(\mu^2+9)^2}\right\}$$

$$=\lambda\left\{\frac{1+\mu^4}{64}+\frac{1}{4}\frac{\mu^4}{(\mu^2+1)^2}+\frac{1}{16}\frac{\mu^4}{(9\mu^2+1)^2}+\frac{1}{16}\frac{\mu^4}{(\mu^2+9)^2}\right\}+\left\{\frac{1+\mu^4}{128}-\frac{5}{32}\frac{\mu^4}{(\mu^2+1)^2}-\frac{1}{32}\frac{\mu^4}{(9\mu^2+1)^2}-\frac{1}{32}\frac{\mu^4}{(\mu^2+9)^2}\right\}$$

$$(\lambda+\tfrac{1}{2})\left\{\left(\frac{1}{8}+\frac{2\mu^4}{(\mu^2+1)^2}\right)\lambda+\left(\frac{3}{32}+\frac{3\mu^4}{2(\mu^2+1)^2}\right)\right\}-(\lambda+\tfrac{3}{2})\left(\frac{1}{32}+\frac{\mu^4}{2(\mu^2+1)^2}\right)$$

$$=\left\{\frac{1}{8}+\frac{2\mu^4}{(\mu^2+1)^2}\right\}\lambda^2+\left\{\frac{1}{32}+\frac{\mu^4}{2(\mu^2+1)^2}\right\}\left\{3\lambda+\frac{3}{2}-\lambda-\frac{3}{2}\right\}+\left\{\frac{1}{16}+\frac{\mu^4}{(\mu^2+1)^2}\right\}\lambda$$

$$=\left\{\frac{1}{8}+\frac{2\mu^4}{(\mu^2+1)^2}\right\}\lambda^2+\left\{\frac{1}{8}+\frac{2\mu^4}{(\mu^2+1)^2}\right\}\lambda$$

$$(\lambda+\tfrac{1}{2})\left[\frac{1}{4}\lambda+\left\{\frac{1}{8}+\frac{\mu^4}{(\mu^2+1)^2}\right\}\right]-(\lambda+\tfrac{3}{2})\left[\frac{\lambda}{4}+\frac{1}{8}\right]$$

$$=\frac{\lambda}{4}\left[\lambda+\frac{1}{2}-\lambda-\frac{3}{2}\right]+\frac{1}{8}\left[\lambda+\frac{1}{2}-\lambda-\frac{3}{2}\right]+(\lambda+\tfrac{1}{2})\frac{\mu^4}{(\mu^2+1)^2}$$

$$=\left\{\frac{\mu^4}{(\mu^2+1)^2}-\frac{1}{4}\right\}\lambda+\left\{\frac{1}{2}\frac{\mu^4}{(\mu^2+1)^2}-\frac{1}{8}\right\}$$

$$(\lambda+\tfrac{1}{2})\left\{(1+\mu^4)\lambda+\frac{1}{4}(1+\mu^2)^2+\frac{1}{2}(1+\mu^4)\right\}-(\lambda+\tfrac{3}{2})\left\{(1+\mu^4)\lambda+\frac{1}{2}(1+\mu^4)\right\}$$

$$=-\left\{(1+\mu^4)\lambda+\frac{1}{2}(1+\mu^4)\right\}+(\lambda+\tfrac{1}{2})\frac{1}{4}(1+\mu^2)^2$$

$$=\left\{\frac{1}{4}(1+\mu^2)^2-(1+\mu^4)\right\}\lambda+\left\{\frac{1}{8}(1+\mu^2)^2-\frac{1}{2}(1+\mu^4)\right\}$$

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$$0 = (\gamma\xi)^2 \Bigg[\left\{ \frac{3\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{(9\mu^2+1)^2} + \frac{\mu^4}{(\mu^2+9)^2} \right\} \lambda^3 + \left\{ \frac{9}{2}\frac{\mu^4}{(\mu^2+1)^2} + \frac{3}{2}\frac{\mu^4}{(9\mu^2+1)^2} + \frac{3}{2}\frac{\mu^4}{(\mu^2+9)^2} \right\} \lambda^2$$

$$+ \left\{ \frac{1+\mu^4}{16} + \frac{1}{4}\frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4}\frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{4}\frac{\mu^4}{(\mu^2+9)^2} \right\} \lambda + \left\{ \frac{1+\mu^4}{32} - \frac{5}{8}\frac{\mu^4}{(\mu^2+1)^2} - \frac{1}{8}\frac{\mu^4}{(9\mu^2+1)^2} - \frac{1}{8}\frac{\mu^4}{(\mu^2+9)^2} \right\} \Bigg]$$

$$-(\gamma\xi) \Bigg[\left\{ \frac{1}{2} + \frac{8\mu^4}{(\mu^2+1)^2} \right\} \lambda^2 + \left\{ \frac{1}{2} + \frac{8\mu^4}{(\mu^2+1)^2} \right\} \lambda \Bigg] + \Bigg[\left\{ \frac{4\mu^4}{(\mu^2+1)^2} - 1 \right\} \lambda + \left\{ \frac{2\mu^4}{(\mu^2+1)^2} - \frac{1}{2} \right\} \Bigg]$$

$$- \frac{2}{3(1-\nu^2)}\gamma^2 \Bigg[\left\{ 2(1+\mu^4) - \frac{1}{2}(1+\mu^2)^2 \right\} \lambda + \left\{ (1+\mu^4) - \frac{1}{4}(1+\mu^2)^2 \right\} \Bigg]$$

$$\frac{\sigma R}{Et}\gamma\mu^2 = (\gamma\xi)^2 \Bigg[\left\{ \frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{4(9\mu^2+1)^2} + \frac{\mu^4}{4(\mu^2+9)^2} \right\} \lambda^2 + \left\{ \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4}\frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{4}\frac{\mu^4}{(\mu^2+9)^2} \right\} \lambda$$

$$+ \left\{ \frac{1+\mu^4}{64} + \frac{1}{4}\frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{16}\frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{16}\frac{\mu^4}{(\mu^2+9)^2} \right\} \Bigg] - (\gamma\xi) \left\{ \frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right\} \left(\lambda + \frac{1}{2} \right)$$

$$+ \left[\frac{\mu^4}{(\mu^2+1)^2} \right] + \frac{1}{3(1-\nu^2)}\gamma^2 \left\{ \frac{1}{4}(1+\mu^2)^2 \right\}$$

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$$A_3 \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0 = 0 \qquad \underline{\underline{141}}$$

$$A_3 = (\gamma\xi)^2 \left\{ \frac{3\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{(9\mu^2+1)^2} + \frac{\mu^4}{(\mu^2+9)^2} \right\}$$

$$A_2 = (\gamma\xi)^2 \left\{ \frac{9}{2}\frac{\mu^4}{(\mu^2+1)^2} + \frac{3}{2}\frac{\mu^4}{(9\mu^2+1)^2} + \frac{3}{2}\frac{\mu^4}{(\mu^2+9)^2} \right\} - (\gamma\xi)\left\{ \frac{1}{2} + \frac{8\mu^4}{(\mu^2+1)^2} \right\}$$

$$A_3 = (\gamma\xi)^2 \left\{ \frac{1+\mu^4}{16} + \frac{1}{4}\frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4}\frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{4}\frac{\mu^4}{(\mu^2+9)^2} \right\} - (\gamma\xi)\left\{ \frac{1}{2} + \frac{8\mu^4}{(\mu^2+1)^2} \right\} + \left\{ \frac{4\mu^4}{(\mu^2+1)^2} - 1 \right\}$$

$$- \frac{2}{3(1-\nu^2)}\gamma^2 \left\{ 2(1+\mu^4) - \frac{1}{2}(1+\mu^2)^2 \right\}$$

$$A_0 = (\gamma\xi)^2 \left\{ \frac{1+\mu^4}{32} - \frac{5}{8}\frac{\mu^4}{(\mu^2+1)^2} - \frac{1}{8}\frac{\mu^4}{(9\mu^2+1)^2} - \frac{1}{8}\frac{\mu^4}{(\mu^2+9)^2} \right\} + \left\{ \frac{2\mu^4}{(\mu^2+1)^2} - \frac{1}{2} \right\}$$

$$- \frac{2}{3(1-\nu^2)}\gamma^2 \left\{ (1+\mu^4) - \frac{1}{4}(1+\mu^2)^2 \right\}$$

$$\frac{\sigma R}{Et} = \left\{ \frac{1}{\gamma} \frac{\mu^2}{(\mu^2+1)^2} + \frac{1}{12} \frac{1}{(1-\nu^2)} \frac{\gamma(\mu^2+1)^2}{\mu^2} \right\}$$

$$+ \frac{1}{\gamma\mu^2} \left[(\gamma\xi)^2 \left[\frac{\mu^4}{(\mu^2+1)^2} + \frac{\mu^4}{4(9\mu^2+1)^2} + \frac{\mu^4}{4(\mu^2+9)^2} \right] \lambda^2 \right.$$

$$+ \left\{ (\gamma\xi)^2 \left[\frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{4} \frac{\mu^4}{(\mu^2+9)^2} \right] - (\gamma\xi) \left[\frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right] \right\} \lambda$$

$$\left. + \left\{ (\gamma\xi)^2 \left[\frac{1+\mu^4}{64} + \frac{1}{4} \frac{\mu^4}{(\mu^2+1)^2} + \frac{1}{16} \frac{\mu^4}{(9\mu^2+1)^2} + \frac{1}{16} \frac{\mu^4}{(\mu^2+9)^2} \right] - \frac{(\gamma\xi)}{2} \left[\frac{1}{8} + \frac{2\mu^4}{(\mu^2+1)^2} \right] \right\} \right]$$

$$\gamma^2 = \left\{ \frac{\mu^2}{(\mu^2+1)^2} \right\} \sqrt{12(1-\nu^2)}$$ for σ_{min} at $\delta/t = 0$

$\boxed{\mu = 1.5}$

$\mu^2 = 0.25, \quad \mu^4 = 0.0625$

$\frac{\mu^4}{(\mu^2+1)^2} = 0.0400000, \quad \frac{\mu^4}{(9\mu^2+1)^2} = 0.005917160, \quad \frac{\mu^4}{(\mu^2+9)^2} = 0.00070460$

$A_3 = 0.12664762\,(\gamma\xi)^2$

$A_2 = 0.18997143\,(\gamma\xi)^2 - 0.8200000\,(\gamma\xi)$

$A_1 = 0.07806816\,(\gamma\xi)^2 - 0.8200000\,(\gamma\xi) - 0.840000 - 0.98443223\,\gamma^2$

$A_0 = 0.007372725\,(\gamma\xi)^2 - 0.42000 - 0.49221612\,\gamma^2$

$$\frac{\sigma R}{Et} = \left\{ \frac{0.16}{\gamma} + 0.5723443\,\gamma \right\} + \frac{4}{\gamma}\left\{ 0.04166191\,(\gamma\xi)^2 \lambda^2 + \left[0.04166191\,(\gamma\xi)^2 - 0.205\,(\gamma\xi) \right]\lambda + \left[0.02701704\,(\gamma\xi)^2 - 0.1025\,(\gamma\xi) \right] \right\}$$

$\gamma_{max} = 0.16\sqrt{12(1-\nu^2)} = 0.52873$

Use 23 waves

$\underline{\gamma = 0.529}$

$\boxed{\mu = 0.5}$ $\quad f = 0.529, \quad f^2 = 0.279841$

$\frac{\sigma R}{Et} = 0.605228 \quad$ (Absolute ϕ_{min} = 0.6052275)

$\mu = 0.5 \quad \gamma = 0.529 \quad \xi = 0.7$

$(\gamma\xi) = 0.3703, \quad (\gamma\xi)^2 = 0.13712209$

$0.017366186\,\lambda^3 - 0.27759672\,\lambda^2 - 1.40842563\,\lambda - 0.55673136 = 0$

$F(-\lambda) = \lambda^3 + 15.98490\,\lambda^2 - 81.10161\,\lambda + 32.058355 = 0$

$F'(-\lambda) = 3\lambda^2 + 31.96980\,\lambda - 81.10161$

$F(0.43328) = +0.000860, \qquad F'(0.43328) = -66.68654$

0000129

$F(0.4332929) = 0.K.$

$\lambda = -0.4332929$

$\frac{\sigma R}{Et} = 0.605228 + \frac{4}{0.529}\{0.00571277\lambda^2 - 0.0701 9873\lambda - 0.03425112\}$

$= \underline{0.58434}$

$\frac{\varepsilon R}{t} = \underline{0.5864}$

$\mu = 0.5 \quad \gamma = 0.529, \quad \xi = 1.5$

$(\gamma\xi) = 0.7935 \quad (\gamma\xi)^2 = 0.62964225$

$0.07974269 2\lambda^3 - 0.53105596\lambda^2 - 1.71699949\lambda - 0.55310042 = 0$

$F(-\lambda) = \lambda^3 + 6.659619\lambda^2 - 21.531747\lambda + 6.936064$

$F'(-\lambda) = 3\lambda^2 + 13.319238\lambda - 21.531747$

$F(0.3657\ \ 0000889) = 0.001446 \qquad F'(0.3657) = 16.2597$

$F(0.3657889) = $ O.K.

$\lambda = -0.3657889$

$\frac{\sigma R}{Et} = 0.605228 + \frac{4}{0.529}\left\{0.02623210\lambda^2 - 0.13643540\lambda - 0.06432268\right\}$

$= \underline{\underline{0.52276}}$

$\frac{\varepsilon R}{t} = \underline{\underline{0.5324}}$

$\Phi =$

$$\mu = 0.5, \quad \gamma = 0.529, \quad \xi = 2.5$$

$$(\gamma\xi) = 1.3225, \qquad (\gamma\xi)^2 = 1.74900625$$

$$0.22150748\,\lambda^3 - 0.75218878\,\lambda^2 - 2.06339280\,\lambda - 0.54484828 = 0$$

$$F(-\lambda) = \lambda^3 + 3.3957714\,\lambda^2 - 9.3152285\,\lambda + 2.4597286$$

$$F'(-\lambda) = 3\lambda^2 + 6.7915428\,\lambda - 9.3152245$$

$$F(0.3\quad) = -0.0022205 \qquad F'(0.3) = 7.00777$$

.0003169

$$F(0.2996831) = 0.0000007$$

0000001

$$\lambda = -0.2996830$$

$$\frac{\sigma R}{Et} = 0.605228 + \frac{4}{0.529}\left\{0.07286694\,\lambda^2 - 0.19824556\,\lambda - 0.08830328\right\}$$

$$= \underline{\underline{0.43624}}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.4641}} \qquad \gamma^2 = 0.279841$$

$$\xi = 0.43624^2 + 6.25\left[0.0010931(-\lambda)^4 - 0.0021862(-\lambda)^3 + 0.0510580(-\lambda)^2 - 0.0330203(-\lambda) + 0.0098605\right]$$

$$= 0.2184 \qquad \Phi = -0.093259$$

$$\frac{\lambda}{E} + \nu\frac{\sigma}{E} - \frac{1}{2}n^2\left\{\frac{1}{16}f_1^2 + \frac{1}{8}\left(\frac{1}{2}f_1+f_2\right)^2\right\} + \left(f_0+\frac{1}{4}f_1\right) = 0 \qquad \text{146 (a)}$$

$$\therefore \quad \frac{\lambda}{E} = \frac{1}{2}n^2\left\{\frac{3}{32}f_1^2 + \frac{1}{8}f_1f_2 + \frac{1}{8}f_2^2\right\} - \left(f_0+\frac{1}{4}f_1\right) - \nu\frac{\sigma}{E}$$

$$\varepsilon = +\left[\frac{\sigma}{E} + \nu\frac{\lambda}{E} + \frac{1}{2}n^2\mu^2\left\{\frac{3}{32}f_1^2 + \frac{1}{8}f_1f_2 + \frac{1}{8}f_2^2\right\}\right]$$

Decrease in potential

$$= -8\frac{\sigma}{E}\left[\frac{\sigma}{E} + \nu\frac{\lambda}{E} + n^2\mu^2\left\{\frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2\right\}\right]$$

$$= -4\left[2(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + n^2\left\{\frac{3}{32}(\mu^2+\nu)f_1^2 + \frac{1}{8}(\mu^2+\nu)f_1f_2 + \frac{1}{8}(\mu^2+\nu)f_2^2\right\}\frac{\sigma}{E}\right.$$

$$\left. - 2\nu\left(f_0+\frac{1}{4}f_1\right)\frac{\sigma}{E}\right]$$

$$4\left[\left(\frac{\lambda}{E}\right)^2 + \left(\frac{\sigma}{E}\right)^2 + 2\nu\frac{\sigma}{E}\frac{\lambda}{E}\right] = 4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + n^4\left\{\frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2\right\}^2\right.$$

$+\left(f_0+\frac{1}{4}f_1\right)^2 - 2n^2\left\{\frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2\right\}\left(f_0+\frac{1}{4}f_1\right)$ ~~$- 2\nu\frac{\sigma}{E}n^2\left\{\frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2\right\}$~~

~~$+ 2\nu\frac{\sigma}{E}\left(f_0+\frac{1}{4}f_1\right)$~~ ~~$+ 2\nu\frac{\sigma}{E}n^2\left\{\frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2\right\}$~~ ~~$- 2\nu\frac{\sigma}{E}\left(f_0+\frac{1}{4}f_1\right)$~~ $\Big]$

$$K = -4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + n^2\left\{\frac{3}{32}(\mu^2+\nu)f_1^2 + \frac{1}{8}(\mu^2+\nu)f_1f_2 + \frac{1}{8}(\mu^2+\nu)f_2^2\right\}\frac{\sigma}{E}\right.$$

$$- 2\nu\left(f_0+\frac{1}{4}f_1\right)\frac{\sigma}{E} - n^4\left\{\frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2\right\}^2 - \left(f_0+\frac{1}{4}f_1\right)^2$$

$$\left. + 2n^2\left(f_0+\frac{1}{4}f_1\right)\left\{\frac{3}{64}f_1^2 + \frac{1}{16}f_1f_2 + \frac{1}{16}f_2^2\right\}\right]$$

$$-2\nu\frac{\sigma}{E}-2\left(f_0+\frac{1}{4}f_1\right)+2n^2\left\{\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right\}=0 \qquad \frac{741}{(a)}$$

$$K=-4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2+n^2\left\{\frac{3}{32}(\mu^2+\nu)f_1^2+\frac{1}{8}(\mu^2+\nu)f_1f_2+\frac{1}{8}(\mu^2+\nu)f_2^2\right\}\frac{\sigma}{E}\right.$$

$$\left.-n^4\left\{\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right\}^2+\left(f_0+\frac{1}{4}f_1\right)^2\right]$$

$$\boxed{K=-4\left(\frac{\sigma}{E}\right)^2-n^2\mu^2\left\{\frac{3}{8}f_1^2+\frac{1}{2}f_1f_2+\frac{1}{2}f_2^2\right\}}$$

$$\varepsilon=(1-\nu^2)\frac{\sigma}{E}-\nu\left(f_0+\frac{1}{4}f_1\right)+n^2(\mu^2+\nu)\left\{\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right\}$$

$$-\nu^2\frac{\sigma}{E}-\nu\left(f_0+\frac{1}{4}f_1\right)+n^2\nu\left\{\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right\}=0$$

$$\varepsilon=\frac{\sigma}{E}+n^2\mu^2\left\{\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right\}$$

$$\boxed{\frac{\varepsilon R}{t}=\frac{\sigma R}{Et}+\frac{1}{16}\mu^2\cdot(1\xi)\xi\left\{\lambda^2+\lambda+0.75\right\}}$$

$\mu = 0.5 \quad \gamma = 0.529, \quad \xi = 4$

$(\gamma\xi) = 2.116, \quad (\gamma\xi)^2 = 4.477456$

$0.56705915\lambda^3 - 0.88453128\lambda^2 - 2.50105775\lambda - 0.52473368 = 0$

$F(-\lambda) = \lambda^3 + 1.5598572\lambda^2 - 4.4105766\lambda + 0.9253597$

$F'(-\lambda) = 3\lambda^2 + 3.1197144\lambda - 4.4105766$

$F(0.232) = -0.0014491 \qquad F'(0.232) = -3.525$

0004111

$F(0.2315889) = +0.0000005$

$\lambda = -0.2315890$

$$\frac{\sigma R}{Et} = 0.605228 + \frac{4}{0.529}\left\{0.11653937\lambda^2 - 0.24724063\lambda - 0.09592239\right\}$$

$= \underline{\underline{0.38852}}$

$\frac{\varepsilon R}{t} = \underline{\underline{0.4642}} \qquad \gamma^2 = 0.279841$

$$\xi = 0.38852^2 + 16\left[0.0027984(-\lambda)^4 - 0.0055968(-\lambda)^3 + 0.0607205(-\lambda)^2 - 0.0308109(-\lambda) + 0.0080752\right]$$

$= 0.477 \qquad \Phi = -0.071801$

$\mu = 0.5, \quad \gamma = 0.529, \quad \xi = 6$

$(\gamma\xi) = 3.174, \qquad (\gamma\xi)^2 = 10.074276$

$1.27588308\lambda^3 - 0.68885538\lambda^2 - 2.93118431\lambda - 0.48347295 = 0$

$F(-\lambda) = \lambda^3 + 0.53990479\lambda^2 - 2.29776879\lambda + 0.37893202$

$F'(-\lambda) = 3\lambda^2 + 1.07980958\lambda - 2.29776879$

$F(0.174 \underset{0003137}{}) = 0.00073443, \qquad F'(0.174) = -2.0191$

$F(0.1743637) = 0.0000024$

$\lambda = -0.17436381$

$$\frac{\sigma R}{Et} = 0.605228 + \frac{4}{0.529}\left\{0.4194358\lambda^2 - 0.23095642\lambda - 0.05315788\right\}$$

$= 0.60427$

$\frac{\varepsilon R}{t} = 0.7846 \qquad \gamma^2 = 0.279841$

$$\xi = 0.60427^2 + 36\left[0.0062964(-\lambda)^4 - 0.0125928(-\lambda)^3 + 0.0805408(-\lambda)^2 - 0.0335775(-\lambda) + 0.0080631\right]$$

$= 0.5306$

$\Phi = -0.208810$

$\mu = 0.5, \quad \gamma = 0.289, \quad \xi = 3$

$\gamma^2 = 0.083521, \quad (\gamma\xi) = 0.867, \quad (\gamma\xi)^2 = 0.751689$

$0.0951996228\lambda^3 - 0.56814057\lambda^2 - 1.57447779\lambda - 0.45556880 = 0$

$F(-\lambda) = \lambda^3 + 5.967887\lambda^2 - 16.537697\lambda + 4.785405 = 0.$

$F'(-\lambda) = 3\lambda^2 + 11.935774\lambda - 16.537697$

$F(0.33) = +0.013475 \qquad F'(0.33) = 12.273$

001098

$F(0.331098) = +0.000007$

$\lambda = -0.331099$

$$\frac{\sigma R}{Et} = 0.7190407 + \frac{4}{0.289}\left\{0.03131680\lambda^2 - 0.14641820\lambda - 0.06855909\right\}$$

$= 0.4886323$

$\frac{\varepsilon R}{t} = \underline{0.5101}$

$$\xi = 0.4886323^2 + 9\left[0.0004698(-\lambda)^4 - 0.0009396(-\lambda)^3 + 0.0379753(-\lambda)^2 - 0.0263970(-\lambda) + 0.0074296\right]$$

$= 0.2842 \qquad \Phi = -0.117151$

$$\boxed{\mu = 0.5, \quad \gamma = 0.289, \quad \xi = 4.5}$$

$$(\gamma\xi) = 1.3005, \qquad (\gamma\xi)^2 = 1.69130025$$

$$0.21419915l\,\lambda^3 - 0.7451112 7\,\lambda^2 - 1.85659407\,\lambda - 0.44864183 = 0$$

$$F(-\lambda) = \lambda^3 + 3.4785912\,\lambda^2 - 8.6676071\,\lambda + 2.0945080 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 6.9571824\,\lambda - 8.6676071$$

$$F(0.27 \underset{004189}{}) = +0.0275264 \qquad F'(0.27) = -6.57046$$

$$F(0.274190 \underset{0000109}{}) = +0.0000715 \qquad F'(0.274190) = -6.53448$$

$$\lambda = -0.2742009$$

$$\frac{\sigma R}{Et} = 0.7190407 + \frac{4}{0.289}\left\{0.07046280\,\lambda^2 - 0.19613970\,\lambda - 0.08760832\right\}$$

$$= \underline{\underline{0.3241919}}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.3746}} \qquad \gamma^2 = 0.083521$$

$$\xi = 0.3241919^2 + 20.25\left[0.00105706(-\lambda)^4 - 0.00211412(-\lambda)^3 + 0.0413028(-\lambda)^2 - 0.0235831(-\lambda) + 0.0057883\right]$$

$$= 0.1535 \qquad \Phi = -0.04469 2$$

$\mu = 0.5, \quad \gamma = 0.49, \quad \xi = 5.5$

$(\beta\xi) = 1.58950, \quad (\beta\xi)^2 = 2.52651025$

$0.31997648\lambda^3 - 0.82342523\lambda^2 - 2.02837076\lambda - 0.44248452 = 0$

$F(-\lambda) = \lambda^3 + 2.5733930\lambda^2 - 6.3391245\lambda + 1.3824158$

$F'(-\lambda) = 3\lambda^2 + 5.1467860\lambda - 6.3391245$

$F(0.245 \; 0002135) = -0.0010457 \qquad F'(0.245) = 4.8981$

$F(0.2447865) = $ o.K.

$\lambda = -0.2447866$

$$\frac{\sigma R}{Et} = 0.7190407 + \frac{4}{0.789}\left\{0.10525424\lambda^2 - 0.22058826\lambda - 0.09466492\right\}$$

$= \underline{0.2434603}$

$\frac{\varepsilon R}{t} = \underline{0.3207}$

$$\xi = 0.2434603^2 + 30.25\left[0.00157907(-\lambda)^4 - 0.00315814(-\lambda)^3 + 0.0442606(-\lambda)^2 - 0.0823161(-\lambda) + 0.0049465\right]$$

$= 0.1227 \qquad \Phi = -0.016728$

$\mu = 0.5 \quad \gamma = 0.289, \quad \xi = 7$

$(\sqrt{\xi}) = 2.023 \qquad (\sqrt{\xi})^2 = 4.092529$

$$0.51830906\lambda^3 - 0.88139641\lambda^2 - 2.26158456\lambda - 0.43093958 = 0$$

$$F(-\lambda) = \lambda^3 + 1.7005229\lambda^2 - 4.3633900\lambda + 0.8314336 = 0$$

$$F'(-\lambda) = 3\lambda^2 + 3.4010458\lambda - 4.3633900$$

$F(0.2098) = +0.0000792 \qquad F'(0.2098) = 3.5178$

0000225

$F(0.2098225) = 0.K.$

$$\lambda = -0.2098225$$

$$\frac{\sigma R}{Et} = 0.7190407 + \frac{4}{0.289}\left\{0.17050257\lambda^2 - 0.24421243\lambda - 0.09648944\right\}$$

$= 0.1925111$

$\dfrac{\varepsilon R}{t} = \underline{\underline{0.3218}} \qquad \gamma^2 = 0.083521$

$$\xi = 0.1925111^2 + 49\left[0.00255783(-\lambda)^4 - 0.00511566(-\lambda)^3 + 0.0498065(-\lambda)^2 - 0.0213290(-\lambda) + 0.0040623\right]$$

$= 0.1222 \qquad \Phi = -0.000850$

134

$$\mu = 0.5, \quad \gamma = 0.289, \quad \xi = 9$$

$$(\gamma\xi) = 2.601, \qquad (\gamma\xi)^2 = 6.765201$$

$$0.85679661\lambda^3 - 0.84762509\lambda^2 - 2.52689397\lambda - 0.41123617 = 0$$

$$F(-\lambda) = \lambda^3 + 0.9892955\lambda^2 - 2.9492342\lambda + 0.4799694$$

$$F'(-\lambda) = 3\lambda^2 + 1.9785910\lambda - 2.9492342$$

$$F(0.175 \ \ ) = -0.0004900 \qquad F'(0.175) = 2.5111$$

0001951

$$F(0.1748049) = \quad 0.K.$$

$$\lambda = -0.1748049$$

$$\frac{\sigma R}{Et} = 0.7190407 + \frac{4}{0.289}\Big/ 0.28185120\lambda^2 - 0.25135380\lambda - 0.08372179\Big\}$$

$$= \underline{0.7861487}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.5077}} \qquad \gamma^2 = 0.083521$$

$$\zeta = 0.7861487^2 + 81\Big[0.00422825(-\lambda)^4 - 0.00845650(-\lambda)^3 + 0.0592715(-\lambda)^2$$
$$- 0.0217180(-\lambda) + 0.0035902\Big]$$

$$= 0.70854 \qquad \Phi = -0.041008$$

$\boxed{\mu = 0.5, \quad \gamma = 0.196, \quad \xi = 5}$ 155

$\gamma^2 = 0.038416 \qquad (\gamma\xi) = 0.98, \quad (\gamma\xi)^2 = 0.9604$

$0.12163237\lambda^3 - 0.62115144\lambda^2 - 1.60644129\lambda - 0.43182874 = 0$

$F(-\lambda) = \lambda^3 + 5.1067938\lambda^2 - 13.2073500\lambda + 3.5502781$

$F'(-\lambda) = 3\lambda^2 + 10.2135876\lambda - 13.2073500$

$F(0.307) = +0.0058663 \qquad F'(0.307) = 9.789$

0005993

$F(0.3075993) = +0.0000019 \qquad F'(0.3075993) = 9.78$

0000002

$F(0.3075995) =$ O.K.

$\lambda = -0.3075995$

$$\frac{\sigma R}{Et} = 0.9285060 + \frac{4}{0.196}\left\{0.04001210\lambda^2 - 0.1608879\lambda - 0.02450243\right\}$$

$= \underline{\underline{0.4952824}}$

$\dfrac{\varepsilon R}{t} = \underline{\underline{0.5364}}$

$$\xi = 0.4952824^2 + 25\left[0.00060025(-\lambda)^4 - 0.0012005(-\lambda)^3 + 0.0365201(-\lambda)^2 - 0.0233636(-\lambda) + 0.0060060\right]$$

$= 0.3014 \qquad \Phi = -0.114969$

$\mu = 0.5 \quad \gamma = 0.196 \quad \xi = 6.5$

251

$\gamma^2 = 0.038416 \qquad (\gamma\xi) = 1.274 \qquad (\gamma\xi)^2 = 1.623076$

$0.20555871\lambda^3 - 0.73634193\lambda^2 - 1.79574739\lambda - 0.42694338 = 0$

$F(-\lambda) = \lambda^3 + 3.5821490\lambda^2 - 8.7311289\lambda + 2.0269900$

$F'(-\lambda) = 3\lambda^2 + 7.1642980\lambda - 8.7361289$

$F(0.27 \quad) = -0.0009431 \qquad F'(0.27) = 6.583$

0001433

$F(0.2698567) = 0.0000003$

$\lambda = -0.2698567$

$\frac{\sigma R}{Et} = 0.9285060 + \frac{4}{0.196}\left\{0.06262045\lambda^2 - 0.19354955\lambda - 0.01673429\right\}$

$= 0.3248460$

$\frac{\varepsilon R}{t} = 0.3964 \qquad \gamma^2 = 0.038416$

$\xi = 0.3248460^2 + 42.25\left[0.0010144225(-\lambda)^4 - 0.0030221845(-\lambda)^3 + 0.0388669(-\lambda)^2 - 0.0245293(-\lambda) + 0.0049236\right]$

$= 0.1862 \qquad \Phi = -0.035669$

$\boxed{\mu = 0.5 \quad \gamma = 0.196, \quad \xi = 8.0}$ 757

$f(\xi) = 1.568, \qquad f(\xi)^2 = 2.458624$

$0.31137888\lambda^3 - 0.81819168\lambda^2 - 1.97163770\lambda - 0.42077358 = 0$

$F(-\lambda) = \lambda^3 + 2.6292460\lambda^2 - 6.3319571\lambda + 1.3513556$

$F'(-\lambda) = 3\lambda^2 + 5.2584920\lambda - 6.3319571$

$F(0.239) = 0.0018549$ $\qquad F'(0.239) = 4.9038$

0003783

$F(0.2393784) = 0.k$

$\lambda = -0.2393784$

$\frac{\sigma R}{Et} = 0.9285060 + \frac{4}{0.196}\{0.10243097\lambda^2 - 0.21900903\lambda - 0.09429526\}$

$= 0.1938126$

$\frac{\varepsilon R}{t} = \underline{0.3051} \qquad \gamma^2 = 0.038416$

$\xi = 0.1938126^2 + 64[0.00153664(-\lambda)^4 - 0.00307328(-\lambda)^3 + 0.0418059(-\lambda)^2 - 0.0201992(-\lambda) + 0.0040502]$

$= 0.1383 \qquad \Phi = +0.010016$

$\mu = 0.5 \quad \gamma = 0.196 \quad \xi = 10$

$(\gamma\xi) = 1.96 \qquad (\gamma\xi)^2 = 3.8416$

$0.48652950\lambda^3 - 0.87740575\lambda^2 - 2.18511131\lambda - 0.41058805 = 0$

$F(-\lambda) = \lambda^3 + 1.8033968\lambda^2 - 4.4912205\lambda + 0.8439119 = 0$

$F'(-\lambda) = 3\lambda^2 + 3.6067936\lambda - 4.4912205$

$F(0.207\ ) = 0.0003727 \qquad F'(0.207) = 3.616$

$\quad 0001031$

$F(0.2071031) = $ O.K.

$\lambda = -0.2071031$

$$\frac{\sigma R}{Et} = 0.9285060 + \frac{4}{0.196}\left\{0.16004839\lambda^2 - 0.24125161\lambda - 0.09711134\right\}$$

$= \underline{0.1085248}$

$\frac{\varepsilon R}{t} = \underline{0.2879} \qquad \gamma^2 = 0.038416$

$$\xi = 0.1085248 + 100\left[0.0024010(-\lambda)^4 - 0.0048020(-\lambda)^3 + 0.0467236(-\lambda)^2 - 0.0192101(-\lambda) + 0.0032107\right]$$

$= 0.13158 \qquad \Phi = +0.034546$

$\mu = 0.5, \quad \gamma = 0.196, \quad \xi = 11$

759

$(\gamma\xi) = 2.156 \qquad (\gamma\xi)^2 = 4.648336$

$0.58870069\,\lambda^3 - 0.88486896\,\lambda^2 - 2.28285091\lambda - 0.40464065$

$F(-\lambda) = \lambda^3 + 1.5030881\,\lambda^2 - 3.8777786\,\lambda + 0.6873453$

$F'(-\lambda) = 3\lambda^2 + 3.0061762\,\lambda - 3.8777786$

$F(0.1935) = +0.0005192 \qquad F'(0.1935) = 3.1838$

1631

$F(0.1936631) = $ o.k.

$\lambda = -0.1936631$

$$\frac{\sigma R}{Et} = 0.9285060 + \frac{4}{0.196}\left\{0.19315856\,\lambda^2 - 0.24832144\lambda - 0.09540572\right\}$$

$= 0.1111228$

$\frac{\varepsilon R}{t} = \underline{0.3312} \qquad \gamma^2 = 0.038416$

$$\zeta = 0.1111228^2 + 121\left[0.0029052\,(-\lambda)^4 - 0.0058104\,(-\lambda)^3 + 0.0495805\,(-\lambda)^2 - 0.0190516\,(-\lambda) + 0.0029302\right]$$

$= 0.14015 \qquad \Phi = +0.033621$

$\mu = 0.5, \quad \gamma = 0.196, \quad \xi = 13$

760

$(\gamma\xi) = 2.548, \qquad (\gamma\xi)^2 = 6.492304$

$0.82223485\,\lambda^3 - 0.85600773\,\lambda^2 - 2.46033572\,\lambda - 0.39104661$

$F(-\lambda) = \lambda^3 + 1.0410745\,\lambda^2 - 2.9922542\,\lambda + 0.4755899$

$F'(-\lambda) = 3\lambda^2 + 2.0821490\,\lambda - 2.9922542$

$F(\ 0.17\ ) = 0.0019067 \qquad F'(0.17) = 2.5516$

0007472

$F(0.1707472) = 0.0000011$

$\lambda = -0.1707476$

$\frac{\sigma R}{Et} = 0.9245060 + \frac{4}{0.196}\{0.27048178\,\lambda^2 - 0.25185822\,\lambda - 0.08576716\}$

$= 0.2167276$

$\frac{\varepsilon R}{t} = 0.5316 \qquad \gamma^2 = 0.038416$

$\xi = 0.2167276^2 + 169\,[0.00405769(-\lambda)^4 - 0.00811538(-\lambda)^3 + 0.0561108(-\lambda)^2 - 0.0194068(-\lambda) + 0.0026479]$

$= 0.2047 \qquad \Phi = -0.012863$

$\mu = 0.5, \quad \gamma = 0.144, \quad \xi = 8$ 761

$\gamma^2 = 0.020736 \qquad (\gamma\xi) = 1.152 \qquad (\gamma\xi)^2 = 1.327104$

$0.16807456\lambda^3 - 0.69252816\lambda^2 - 1.70144862\lambda - 0.42042296 = 0$

$F(-\lambda) = \lambda^3 + 4.1203628\lambda^2 - 10.1231776\lambda + 2.5014075$

$F'(-\lambda) = 3\lambda^2 + 8.2407256\lambda - 10.1231776$

$F(0.282) = -0.0032351 \qquad F'(0.282) = 7.561$

0004279

$F(0.2815721) = +0.0000011$

$\lambda = 0.2815722$

$$\frac{\sigma R}{Et} = 1.1935287 + \frac{4}{0.144}\left\{0.05524969\lambda^2 - 0.16087031\lambda - 0.08222558\right\}$$

$= 0.4459175$

$\frac{\varepsilon R}{t} = 0.5248 \qquad \gamma^2 = 0.020736$

$$\xi = 0.4459175^2 + 64\Big[0.00082944(-\lambda)^4 - 0.00165888(-\lambda)^3 - 0.0369586(-\lambda)^2 - 0.0213692(-\lambda) + 0.0049743\Big]$$

$= 0.3176 \qquad \Phi = -0.075218$

$$\boxed{\mu = 0.5 \qquad \gamma = 0.144 \qquad \xi = 10}$$

$$(\gamma\xi) = 1.44 \qquad (\gamma\xi)^2 = 2.0736$$

$$0.26261650\,\lambda^3 - 0.78687524\,\lambda^2 - 1.87933105\,\lambda - 0.41491966 = 0$$

$$F(-\lambda) = \lambda^3 + 4.8724623\,\lambda^2 - 11.6371302\,\lambda + 2.5192515$$

$$F'(-\lambda) = 3\lambda^2 + 9.7449246\,\lambda - 11.6371302$$

$$F(0.248) = -0.0018279 \qquad F'(0.248) = 9.0359$$

0002022

$$F(0.2477978) = -0.0022205$$

$$\lambda = -0.2477977$$

$$\frac{\sigma R}{Et} = 1.1935287 + \frac{4}{0.144}\left\{0.08639014\,\lambda^2 - 0.20880986\,\lambda - 0.09157747\right\}$$

$$= 0.2343567$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.3612}} \qquad \gamma^2 = 0.020736$$

$$\xi = 0.2343567^2 + 100\Big[0.0012960(-\lambda)^4 - 0.0025920(-\lambda)^3 + 0.0396022(-\lambda)^2 - 0.0198562(-\lambda) + 0.0040316\Big]$$

$$= 0.10577 \qquad \Phi = +0.014909$$

163

$\mu = 0.5$ $\gamma = 0.144$ $\xi = 13$

$(\gamma\xi) = 1.872$ $(\gamma\xi)^2 = 3.504384$

$0.44382189\lambda^3 - 0.86930716\lambda^2 - 2.12187238\lambda - 0.40437118 = 0$

$F(-\lambda) = \lambda^3 + 1.9586847\lambda^2 - 4.7809097\lambda + 0.9111125$

$F'(-\lambda) = 3\lambda^2 + 3.9173694\lambda^2 - 4.7809097$

$F(0.2105) = +0.0008481$ $F'(0.2105) = 3.823$

2218

$F(0.2107219) =$ O.K.

$\lambda = -0.2107219$

$\frac{\sigma R}{Et} = 1.1935287 + \frac{4}{0.144}\{0.14599933\lambda^2 - 0.23776067\lambda - 0.09720192\}$

$= 0.0652615$

$\frac{\varepsilon R}{t} = \underline{0.2872}$ $\gamma^2 = 0.020736$

$\xi = 0.0652615^2 + 169[0.00219024(-\lambda)^4 - 0.00430048(-\lambda)^3 + 0.0446692(-\lambda)^2$
$- 0.0184940(-\lambda) + 0.0029936]$

$= 0.1806$ $\Phi = +0.071557$

$\mu = 0.5 \quad \gamma = 0.144 \quad \xi = 16$

$(\gamma\xi) = 2.304, \qquad (\gamma\xi)^2 = 5.308416$

$0.67229825\lambda^3 - 0.88083262\lambda^2 - 2.33527492\lambda - 0.39107205$

$F(-\lambda) = \lambda^3 + 1.3101814\lambda^2 - 3.4735699\lambda + 0.5816943$

$F'(-\lambda) = 3\lambda^2 + 2.6203628\lambda - 3.4735699$

$F(0.1815) = +0.0003807 \qquad F'(0.1815) = 2.899$

1313

$F(0.1816313) =$ o.k.

$\lambda = -0.1816313$

$$\frac{\sigma R}{Et} = 1.1935287 + \frac{4}{0.144}\left\{0.22115875\lambda^2 - 0.25116125\lambda - 0.09274231\right\}$$

$= 0.0872079$

$\frac{\varepsilon R}{t} = 0.4336 \qquad \gamma^2 = 0.020736$

$$\xi = 0.0872079^2 + 256\left[0.00331776(-\lambda)^4 - 0.00663552(-\lambda)^3 + 0.0510580(-\lambda)^2 - 0.0182203(-\lambda) + 0.0026068\right]$$

$= 0.1985 \qquad \Phi = +0.061437$

765

$$\mu=0.5, \quad \gamma=0.144, \quad \xi=19$$

$$(\gamma\xi)=2.736, \qquad (\gamma\xi)^2=7.485696$$

$$0.94804558\lambda^3-0.82145163\lambda^2-2.51953817\lambda-0.37502077=0$$

$$F(-\lambda)=\lambda^3+0.8664685\lambda^2-2.6576134\lambda+0.3955725=0$$

$$F'(-\lambda)=3\lambda^2+1.7329370\lambda-2.6576134$$

$$F(0.1585)=0.0000903, \qquad F'(0.1585)=2.3076$$

$$F(0.1585391)=0.\text{K}.$$

$$\lambda=-0.1585391$$

$$\frac{\sigma R}{Et}=1.1935287+\frac{4}{0.144}\Big\{0.31186839\lambda^2-0.24901161\lambda-0.07819865\Big\}$$

$$=\underline{0.3351987}$$

$$\frac{\varepsilon R}{t}=\underline{\underline{0.8365284}} \qquad \gamma^2=0.020736$$

$$\xi=0.3356987^2+361\Big[0.00467856(-\lambda)^4-0.00935712(-\lambda)^3+0.0587686(-\lambda)^2$$
$$-0.0190351(-\lambda)+0.0022712\Big]$$

$$=0.3640 \qquad \Phi=-0.098821$$

766

$\mu = 0.5, \quad \gamma = 0.121, \quad \xi = 11$

$\gamma^2 = 0.014641, \quad (\gamma\xi) = 1.331, \quad (\gamma\xi)^2 = 1.771561$

$0.22436398\,\lambda^3 - 0.75487402\,\lambda^2 - 1.80753056\,\lambda - 0.41414629$

$F(-\lambda) = \lambda^3 + 3.3645063\,\lambda^2 - 8.0562422\,\lambda + 1.8458680$

$F'(-\lambda) = 3\lambda^2 + 6.7290126\,\lambda - 8.0562422$

$F(0.26) = -0.0037384 \qquad F'(0.26) = 6.1039$

0006124

$F(0.2593876) = +0.0000012$

$\lambda = -0.2593878$

$$\frac{\sigma R}{Et} = 1.3915677 + \frac{4}{0.121}\left\{0.0738066\,\lambda^2 - 0.1990439\,\lambda - 0.0885517\right\}$$

$= \underline{0.3347549}$

$\frac{ER}{t} = \underline{0.4624} \qquad \gamma^2 = 0.014641$

$$\xi = 0.3347549^2 + 121\left[0.0011072\,(-\lambda)^4 - 0.0022144\,(-\lambda)^3 + 0.0382361\,(-\lambda)^2 - 0.0200754\,(-\lambda) + 0.0042362\right]$$

$= 0.30178 \qquad \Phi = -0.003901$

$\mu = 0.5, \quad \gamma = 0.121, \quad \xi = 14$

$(\gamma\xi) = 1.694, \qquad (\gamma\xi)^2 = 2.869136$

$0.36343257\lambda^3 - 0.84393115\lambda^2 - 2.01946587\lambda - 0.40605109$

$F(-\lambda) = \lambda^3 + 2.3221121\lambda^2 - 5.5566454\lambda + 1.1172617$

$F'(-\lambda) = 3\lambda^2 + 4.6442242\lambda - 5.5566454$

$F(0.22) = +0.0178429 \qquad F'(0.22) = -4.3897$

004065

$F(0.2240647) = 0.0000494 \qquad F'(0.2240647) = -4.3654$

113

$F(0.2240760) =$ o.k.

$\lambda = -0.2240760$

$\frac{\sigma R}{Et} = 1.3915677 + \frac{4}{0.121}\{0.11955452\lambda^2 - 0.22721548\lambda - 0.09610593\}$

$= 0.0997487$

$\frac{\varepsilon R}{t} = 0.3132 \qquad \gamma^2 = 0.014641$

$\zeta = 0.0997489^2 + 196\left[0.0017935(-\lambda)^4 - 0.0035870(-\lambda)^3 + 0.0421248(-\lambda)^2 - 0.0186269(-\lambda) + 0.0032380\right]$

$= 0.2341 \qquad \Phi = +0.085809$

$\mu = 0.5$, $\gamma = 0.121$, $\xi = 17$

$(\gamma\xi) = 2.057$, $(\gamma\xi)^2 = 4.231249$

$0.53587762\,\lambda^3 - 0.88292358\,\lambda^2 - 2.21082725\,\lambda - 0.39601305$

$F(-\lambda) = \lambda^3 + 1.6476217\,\lambda^2 - 4.1256196\,\lambda + 0.7389990$

$F'(-\lambda) = 3\lambda^2 + 3.2952434\,\lambda - 4.1256196$

$F(0.1964) = -0.0001433$ $F'(0.1964) = 3.313$

0000426

$F(0.1963574) = $ o.k.

$\lambda = -0.1963574$

$$\frac{\sigma R}{Et} = 1.3915677 + \frac{4}{0.121}\left\{0.17628192\,\lambda^2 - 0.24540308\,\lambda - 0.09652118\right\}$$

$$= \underline{0.0182385}$$

$$\frac{\varepsilon R}{t} = \underline{0.3418104} \qquad \gamma^2 = 0.014641$$

$$\xi = 0.0182385^2 + 289\left[0.0026445(-\lambda)^4 - 0.0052891(-\lambda)^3 + 0.0469468(-\lambda)^2 - 0.0179470(-\lambda) + 0.0025515\right]$$

$= 0.23396$ $\Phi = +0.110746$

$$\boxed{\mu = 0.5, \quad \gamma = 0.121, \quad \xi = 20}$$

769

$$(\gamma\xi) = 2.42, \qquad (\gamma\xi)^2 = 5.8564$$

$$0.74169912\,\lambda^3 - 0.87185132\,\lambda^2 - 2.38161470\,\lambda - 0.38403216 = 0$$

$$F(-\lambda) = \lambda^3 + 1.1754784\,\lambda^2 - 3.2110254\,\lambda + 0.5177735$$

$$F'(-\lambda) = 3\lambda^2 + 2.3509568\,\lambda - 3.2110254$$

$$F(0.174) = -0.0000881, \qquad F'(0.174) = 2.7111$$

0000325

$$F(0.1739675) = 0.\text{K}$$

$$\lambda = -0.1739675$$

$$\frac{\sigma R}{Et} = 1.3915677 + \frac{4}{0.121}\left\{0.24398881\,\lambda^2 - 0.25211119\,\lambda - 0.08982741\right\}$$

$$= \underline{0.1160133}$$

$$\frac{\varepsilon R}{t} = \underline{0.5745756} \qquad \gamma^2 = 0.014641$$

$$\zeta = 0.01347069 + 400\left\{0.00366025\,(-\lambda)^4 - 0.0073205\,(-\lambda)^3 + 0.0527019479\,(-\lambda)^2\right.$$

$$\left. - 0.0180354479\,(-\lambda) + 0.0021977600\right\}$$

$$= \underline{0.2615} \qquad \Phi = +\,0.064063$$

$$\mu = 0.5, \quad \gamma = 0.1, \quad \xi = 13$$

$$(\gamma\xi) = 1.3, \quad (\gamma\xi)^2 = 1.69$$

$$0.21403448\lambda^3 - 0.74494828\lambda^2 - 1.78390913\lambda - 0.41246319 = 0$$

$$F(-\lambda) = \lambda^3 + 3.4805059\lambda^2 - 8.3346811\lambda + 1.9270876$$

$$F'(-\lambda) = 3\lambda^2 + 6.9610118\lambda - 8.3346811$$

$F(0.262) = +0.0003017$ $\qquad\qquad F'(0.265) = 6.3050$

0000479

$$F(0.2620479) = 0.00000$$

$$\lambda = -0.2620479$$

$$\frac{\sigma R}{Et} \doteq 1.6572344 + 40\{0.0704086\lambda^2 - 0.1960913 7\lambda - 0.0875 9120\}$$

$$= \underline{0.4023956}$$

$$\frac{\varepsilon R}{t} = \underline{0.5494}$$

$$\xi = 0.4023956^2 + 169[0.00105625(-\lambda)^4 - 0.0021125(-\lambda)^3 + 0.0377215(-\lambda)^2$$
$$- 0.0200090(-\lambda) + 0.0042383]$$

$= 0.4243$ $\qquad\qquad \Phi = -0.008926$

$\boxed{\mu = 0.5, \quad \gamma = 0.1, \quad \xi = 16}$

$(\gamma\xi) = 1.6, \qquad (\gamma\xi)^2 = 2.56$

$0.32421791\lambda^3 - 0.82567314\lambda^2 - 1.96191983\lambda - 0.40604940 = 0$

$F(-\lambda) = \lambda^3 + 2.5466611\lambda^2 - 6.0514541\lambda + 1.2523966$

$F'(-\lambda) = 3\lambda^2 + 5.0933222\lambda - 6.0514541$

$F(0.23) = 0.0074475 \qquad F'(0.23) = 4.72129$

0015774

$F(0.2315774) = 0.0000082$

17

$\lambda = -0.2315791$

$$\frac{\sigma R}{Et} = 1.6572344 + 40\{0.10665449\lambda^2 - 0.22134551\lambda - 0.09483638\}$$

$= \underline{0.1429292}$

$\frac{\varepsilon R}{t} = \underline{\underline{0.3717}}$

$$\xi = 0.1429292^2 + 256\left[0.0016000(-\lambda)^4 - 0.0032000(-\lambda)^3 + 0.0408025(-\lambda)^2 - 0.0187025(-\lambda) + 0.0033600\right]$$

$= 0.3251 \qquad \Phi = +0.109423$

$\mu = 0.5, \quad \gamma = 0.1, \quad \xi = 19$

$(\beta) = 1.9 \qquad (\beta)^2 = 3.61$

$0.45719791\lambda^3 - 0.87220314\lambda^2 - 2.12601826\lambda - 0.39830813$

$F(-\lambda) = \lambda^3 + 1.9077146\lambda^2 - 4.6501050\lambda + 0.8711952$

$F'(-\lambda) = 3\lambda^2 + 3.8154292\lambda - 4.6501050$

$F(0.206) = +0.0029712 \qquad F'(0.206) = 3.7318$

0007951

$F(0.2067951) = +0.0000016$

$\lambda = -0.2067955$

$\frac{\sigma R}{Et} = 1.6572344 + 40\{0.15039950\lambda^2 - 0.23910050\lambda - 0.09721849\}$

$= \underline{0.0035608}$

$\frac{\varepsilon R}{t} = \underline{\underline{0.3341}}$

$\xi = 0.0035608^2 + 361\left[0.0022563(-\lambda)^4 - 0.0045125(-\lambda)^3 + 0.0445210(-\lambda)^2 - 0.0179210(-\lambda) + 0.0027154\right]$

$= 0.3168 \qquad \Phi = +0.157210$

$\mu = 0.5, \quad \gamma = 0.1, \quad \xi = 22$

$(\gamma\xi) = 2.2 \qquad (\gamma\xi)^2 = 4.84$

$0.61297448\lambda^3 - 0.88453827\lambda^2 - 2.27599443\lambda - 0.38924086$

$F(-\lambda) = \lambda^3 + 1.4430263\lambda^2 - 3.7130330\lambda + 0.6350034$

$F'(-\lambda) = 3\lambda^2 + 2.8860526\lambda - 3.7130330$

$F(0.186) = +0.0007370 \qquad F'(0.186) = 3.072$

$F(0.1862399) = 0.K.$

$\lambda = -0.1862399$

$$\frac{\sigma R}{Et} = 1.65272344 + 40\left\{0.20164364\lambda^2 - 0.24935636\lambda - 0.09473753\right\}$$

$= \underline{0.0051000}$

$\frac{\varepsilon R}{t} = \underline{0.4577}$

$$\xi = 0.0051^2 + 4.84\left[0.003025(-\lambda)^4 - 0.00605(-\lambda)^3 + 0.0488769(-\lambda)^2 - 0.0176644(-\lambda) + 0.0022803\right]$$

$= 0.3148$

$\Phi = +0.155066$

$\mu = 0.5 \quad \gamma = 0.1 \qquad \xi = 25$

$(\gamma\xi) = 2.5, \qquad (\gamma\xi)^2 = 6.25$

$0.7915473\lambda^3 - 0.8626785 6\lambda^2 - 2.41191832\lambda - 0.37884610$

$F(-\lambda) = \lambda^3 + 1.0898631\lambda^2 - 3.0470919\lambda + 0.4788144$

$F'(-\lambda) = 3\lambda^2 + 2.1797262\lambda - 3.0470919$

$F(0.169) = -\ 0.0003897 \qquad F'(0.169) = 2.593$

0001503

$F(0.1688497) =$ o.k.

$\lambda = -\ 0.1688497$

$\frac{\sigma R}{Et} = 1.6572344 + 40\left\{0.26038694\lambda^2 - 0.25211306\lambda - 0.08739350\right\}$

$= \underline{0.1612104}$

$\frac{\varepsilon R}{t} = \underline{0.7565620} \qquad \gamma^2 = 0.01$

$\zeta = 0.0259888 + 625\left\{0.00390625\,(-\lambda)^4 - 0.0078125\,(-\lambda)^3 + 0.0538700496(-\lambda)^2\right.$

$\left. - 0.0179325496\,(-\lambda) + 0.002063105 8\right\}$

$= \underline{0.36137} \qquad \Phi = +\ 0.058716$

$$\boxed{\mu = 0.5, \quad \gamma = 0.081, \quad \xi = 17}$$

$\gamma^2 = 0.006561$, $(\gamma\xi) = 1.377$, $(\gamma\xi)^2 = 1.896129$

$$0.24014023\lambda^3 - 0.76892966\lambda^2 - 1.82757156\lambda - 0.40925084$$

$$F(-\lambda) = \lambda^3 + 3.2020026\lambda^2 - 7.6104347\lambda + 1.7042161$$

$$F'(-\lambda) = 3\lambda^2 + 6.4040052\lambda - 7.6104347$$

$F(0.253) = -0.0000726$ $\quad$ $F'(0.253) = 5.798$

0000125

$F(0.2529875) = $ o.k.

$$\lambda = -0.2529875$$

$$\frac{\sigma R}{Et} = 2.0216685 + \frac{40}{0.81}\left\{0.07899636\lambda^2 - 0.20328864\lambda - 0.08991471\right\}$$

$$= \underline{0.3708413}$$

$$\frac{\varepsilon R}{t} = \underline{\underline{0.5760}}$$

$$\xi = 0.14033117 + 289\left\{0.0011850106(-\lambda)^4 - 0.0023701613(-\lambda)^3 + 0.0382840805(-\lambda)^2\right.$$

$$\left. - 0.0194561874(-\lambda) + 0.0739216599\right\}$$

$= \underline{0.5491}$ $\quad$ $\Phi = +0.061195$

$\mu = 0.5, \quad \gamma = 0.081, \quad \xi = 20$

776

$(\gamma\xi) = 1.62, \quad (\gamma\xi)^2 = 2.6244$

$0.33237401\lambda^3 - 0.82983898\lambda^2 - 1.96997678\lambda - 0.40388191$

$F(-\lambda) = \lambda^3 + 2.4967023\lambda^2 - 5.9269880\lambda + 1.2151429$

$F'(-\lambda) = -3\lambda^2 + 4.9934046\lambda - 5.9269880$

$F(0.23 \quad) = -0.0038218 \qquad F'(0.23) = 4.6198$

0008273

$F(0.2291727) = +0.0000023$

$\lambda = -0.2291732$

$\frac{\sigma R}{Et} = 2.0216615 + \frac{40}{0.81}\{0.10933752\lambda^2 - 0.22242248\lambda - 0.09514648\}$

$= \underline{0.1277009}$

$\frac{\varepsilon R}{t} = \underline{0.4180}$

$\varepsilon = 0.0163075 + 400\{0.00164025(-\lambda)^4 - 0.0032805(-\lambda)^3 + 0.04086315\,69(-\lambda)^2$

$- 0.0184666569(-\lambda) + 0.0032452910\}$

$= \underline{0.4661} \qquad \Phi = +0.179671$

$$\mu=0.5, \quad \gamma=0.081, \quad \xi=24$$

777

$(\gamma\xi) = 1.944 \qquad (\gamma\xi)^2 = 3.779136$

$$0.47861858\lambda^3 - 0.87615213\lambda^2 - 2.14550867\lambda - 0.39536900 = 0$$

$$F(-\lambda) = \lambda^3 + 1.8305853\lambda^2 - 4.4827108\lambda + 0.8260628$$

$$F'(-\lambda) = 3\lambda^2 + 3.6611706\lambda - 4.4827108$$

$F(0.203) = -0.0001255$ 0000122 $\qquad F'(0.203) = 3.116$

$F(0.2029553) =$ o.K.

$$\lambda = -0.2029653$$

$$\frac{\sigma R}{Et} = 2.0216685 + \frac{40}{0.81}\left\{0.15744602\lambda^2 - 0.24107398\lambda - 0.09715893\right\}$$

$$= -0.0397291$$

$$\frac{\varepsilon R}{t} = \underline{0.38909}$$

$$\mathcal{E} = 0.0015784 + 576\left\{0.00236196(-\lambda)^4 - 0.0047239200(-\lambda)^3 + 0.0449525033(-\lambda)^2\right.$$

$$\left. - 0.0176830433(-\lambda) + 0.0025655489\right\}$$

$= \underline{0.4582485}$ $\qquad \Phi = +\ 0.264583$

$$4\left[\left(\frac{\lambda}{E}\right)^2+\left(\frac{\sigma}{E}\right)^2+2\nu\frac{\sigma}{E}\frac{\lambda}{E}\right]$$

$$= 4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2+n^4\left\{\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right\}^2+\left(f_0+\frac{1}{4}f_1\right)\left\{\left(f_0+\frac{1}{4}f_1\right)\right.\right.$$

$$\left.\left.-2n^2\left(\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right)\right\}\right]$$

$$= 4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2+n^4\left\{\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right\}^2+\left\{-\nu\frac{\sigma}{E}+n^2\left(\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right)\right\}\right.$$

$$\left.\left\{\nu\frac{\sigma}{E}+n^2\left(\frac{3}{64}f_1^2+\frac{1}{16}f_1f_2+\frac{1}{16}f_2^2\right)\right\}\right]$$

$$= 4\left(\frac{\sigma}{E}\right)^2$$

$$
\frac{\text{Elastic Strain Energy}}{\frac{1}{2}Et\cdot \text{Area}}\left(\frac{R}{t}\right)^2=\left(\frac{\sigma R}{Et}\right)^2+\xi^2\Bigg[(\gamma\xi)^2\Bigg\{\Bigg[\underset{0.00048828125}{\frac{1+\mu^4}{2048}}+\overset{0.0166015625}{\frac{17}{1024}}\frac{\mu^4}{(1+\mu^2)^2}+\overset{0.00390625}{\frac{1}{256}}\frac{\mu^4}{(9\mu^2+1)^2}+\frac{1}{256}\frac{\mu^4}{(\mu^2+9)^2}\Bigg]
$$

$$
+\Bigg[\overset{0.0703125}{\frac{9}{128}}\frac{\mu^4}{(\mu^2+1)^2}+\overset{0.015625}{\frac{1}{64}}\frac{\mu^4}{(9\mu^2+1)^2}+\overset{0.015625}{\frac{1}{64}}\frac{\mu^4}{(\mu^2+9)^2}\Bigg]\lambda+\Bigg[\overset{0.0859375}{\frac{11}{128}}\frac{\mu^4}{(\mu^2+1)^2}+\overset{0.015625}{\frac{1}{64}}\frac{\mu^4}{(9\mu^2+1)^2}+\overset{0.015625}{\frac{1}{64}}\frac{\mu^4}{(\mu^2+9)^2}\Bigg]\lambda^2
$$

$$
+\overset{0.03125}{\frac{1}{32}}\frac{\mu^4}{(\mu^2+1)^2}\lambda^3+\overset{0.015625}{\frac{1}{64}}\frac{\mu^4}{(\mu^2+1)^2}\lambda^4\Bigg\}
$$

$$
-(\gamma\xi)\Bigg\{\Bigg[\overset{0.00390625}{\frac{1}{256}}+\overset{0.0625}{\frac{1}{16}}\frac{\mu^4}{(\mu^2+1)^2}\Bigg]+\Bigg[\overset{0.0078125}{\frac{1}{128}}+\overset{0.125}{\frac{1}{8}}\frac{\mu^4}{(\mu^2+1)^2}\Bigg]\lambda\Bigg\}+\Bigg\{\Bigg[\overset{0.0078125}{\frac{1}{128}}+\overset{0.0625}{\frac{1}{16}}\frac{\mu^4}{(\mu^2+1)^2}\Bigg]+\overset{0.03125}{\frac{1}{32}}\lambda+\overset{0.03125}{\frac{1}{32}}\lambda^2\Bigg\}
$$

$$
+\overset{0.04578754}{\frac{1}{24(1-\nu^2)}}\gamma^2\Bigg\{(1+\mu^4)\lambda^2+(1+\mu^4)\lambda+\frac{1}{8}(1+\mu^2)^2+\frac{1}{4}(1+\mu^4)\Bigg\}\Bigg]
$$

179

$\mu = 0.5, \quad \gamma = 0.081, \quad \xi = 27$

$(\gamma\xi) = 2.187, \qquad (\gamma\xi)^2 = 4.782969$

$0.60575164\,\lambda^3 - 0.88471254\,\lambda^2 - 2.26640127\,\lambda - 0.38796857$

$F(-\lambda) = \lambda^3 + 1.4605203\,\lambda^2 - 3.7414695\,\lambda + 0.6404747$

$F'(-\lambda) = 3\lambda^2 + 2.9210406\,\lambda - 3.7414695$

$F(0.1863) = +\,0.0005963 \qquad F'(0.1863) = 3.0932$

0001928

$F(0.1864928) = $ o.k.

$\lambda = -0.1864928$

$$\frac{\sigma R}{Et} = 2.0216685 + \frac{40}{0.81}\left\{0.19926762\,\lambda^2 - 0.24906738\,\lambda - 0.09494584\right\}$$

$$= -0.0309795$$

$$\frac{\varepsilon R}{t} = 0.5210242$$

$$\zeta = 0.0009597 + 729\left\{0.0029893556\,(-\lambda)^4 - 0.0059787113\,(-\lambda)^3 + 0.0485074465\,(-\lambda)^2 - 0.0174971533\,(-\lambda) + 0.0022222922\right\}$$

$$= 0.4464547 \qquad \Phi = +\,0.239369$$

$$\boxed{\mu = 0.5, \quad \gamma = 0.081, \quad \xi = 30}$$

781

$$(\gamma\xi) = 2.43, \qquad (\gamma\xi)^2 = 5.9049$$

$$0.74784153\lambda^3 - 0.87083770\lambda^2 - 2.37807418\lambda - 0.37969750$$

$$F(-\lambda) = \lambda^3 + 1.16446882\lambda^2 - 3.1799173\lambda + 0.5077245$$

$$F'(-\lambda) = 3\lambda^2 + 2.3289364\lambda - 3.1799173$$

$$F(0.172) = 0.0003168 \qquad\qquad F'(0.172) = 2.691$$

0001177

$$F(0.1721177) = 0.K$$

$$\lambda = -0.1721177$$

$$\frac{\sigma R}{Et} = 2.0216685 + \frac{40}{0.81}\left\{0.24600941\lambda^2 - 0.25214059\lambda - 0.08954208\right\}$$

$$= \underline{0.1028381}$$

$$\frac{\varepsilon R}{t} = \underline{0.7948} \qquad\qquad \gamma^2 = 0.006561$$

$$\varepsilon = 0.0105758 + 900\left\{0.0036905625(-\lambda)^4 - 0.007381125(-\lambda)^3\right.$$

$$\left. + 0.0524806182(-\lambda)^2 - 0.0176556807(-\lambda) + 0.0020217957\right\}$$

$$= \underline{0.41350620}$$

for $\mu = 1$

$$\mathcal{E} = \frac{S\left(\frac{R}{t}\right)^2}{\frac{1}{2}Et\cdot A} = \left(\frac{\sigma R}{Et}\right)^2 + \xi^2\Big[(\gamma\xi)^2\{0.00390625\,\lambda^4 + 0.00781250\,\lambda^3$$

$$+ 0.021796875\,\lambda^2 + 0.017890625\,\lambda + 0.005205079\}$$

$$-(\gamma\xi)\{0.03906250\,\lambda + 0.01953125\} + \{0.0312500\,\lambda^2 + 0.031250\,\lambda + 0.0234375\}$$

$$+ \gamma^2\{0.09157509\,\lambda^2 + 0.09157509\,\lambda + 0.04578755\}\Big]$$

$$= \left(\frac{\sigma R}{Et}\right)^2 + \xi^2\Big[0.00390625(\gamma\xi)^2(-\lambda)^4 - 0.00781250(\gamma\xi)^2(-\lambda)^3$$

$$+ \{0.021796875(\gamma\xi)^2 + 0.03125000 + 0.09157509\gamma^2\}(-\lambda)^2$$

$$- \{0.017890625(\gamma\xi)^2 - 0.03906250(\gamma\xi) + 0.031250 + 0.09157509\gamma^2\}(-\lambda)$$

$$+ \{0.005205079(\gamma\xi)^2 - 0.01953125(\gamma\xi) + 0.02343750 + 0.04578755\gamma^2\}\Big]$$

$\mu = 0.5$

363

$$\mathcal{E} = \left(\frac{\sigma R}{Et}\right)^2 + \xi^2 \Big[(\gamma\xi)^2 \{ 0.0012088286 + 0.0029113691\lambda + 0.0035413691\lambda^2$$

$$+ 0.00125\lambda^3 + 0.000625\lambda^4 \} - (\gamma\xi) \{ 0.00640625 + 0.0128125\lambda \}$$

$$+ \{ 0.0103125 + 0.03125\lambda + 0.03925\lambda^2 \} + \gamma^2 \{ 0.048649268\lambda^2 + 0.048649268\lambda$$

$$+ 0.021105198 \}$$

$$\mathcal{E} = \left(\frac{\sigma R}{Et}\right)^2 + \xi^2 \Big[0.000625(\gamma\xi)^2(-\lambda)^4 - 0.00125(\gamma\xi)^2(-\lambda)^3$$

$$+ \{ 0.0035413691(\gamma\xi)^2 + 0.03125 + 0.048649268\gamma^2 \}(-\lambda)^2$$

$$- \{ 0.0029163591(\gamma\xi)^2 - 0.0128125(\gamma\xi) + 0.03125 + 0.048649268\gamma^2 \}(-\lambda)$$

$$+ \{ 0.0012088286(\gamma\xi)^2 - 0.00640625(\gamma\xi) + 0.0103125 + 0.021105198\gamma^2 \} \Big]$$

$\cos\frac{\pi x}{R}\cos\frac{\pi y}{R}$

$\frac{\pi x}{R}$	$\frac{\pi y}{R}=0$: $\cos\frac{\pi x}{R}$	$\frac{\pi y}{R}=15°$: $\frac{w}{R}$	$\frac{\pi y}{R}=30°$: $\frac{w}{R}$	$\frac{\pi y}{R}=45°$: $\frac{w}{R}$	$\frac{\pi y}{R}=60°$: $\frac{w}{R}$	$\frac{\pi y}{R}=75°$: $\frac{w}{R}$	
0	1.0000	0.9659	0.8660	0.7071	0.50000	0.2588	
15°	0.9659	0.9330	0.8365	0.6829	0.4830	0.2500	
30°	0.8660	0.8365	0.7500	0.6123	0.4330	0.2241	
45°	0.7071	0.6830	0.6123	0.50000	0.3536	0.1830	
60°	0.5000	0.4830	0.4330	0.3536	0.2500	0.1294	
75°	0.2588	0.2500	0.2241	0.1830	0.1294	0.0670	
90°	0	0	0	0	0	0	

$$\frac{w_1}{R} = \cos^2\frac{\pi(x+y)}{2R}\cos^2\frac{\pi(x-y)}{2R}$$

$\frac{\pi x}{R}$	$\frac{\pi y}{R}=0$: $\frac{w_1}{R}$	$\cos^2\frac{\pi x}{2R}$	$\frac{\pi y}{R}=15°$: $\frac{w_1}{R}$	$\frac{\pi y}{R}=30°$: $\frac{w_1}{R}$	$\frac{\pi y}{R}=45°$: $\frac{w_1}{R}$	$\frac{\pi y}{R}=60°$: $\frac{w_1}{R}$	$\frac{\pi y}{R}=75°$: $\frac{w_1}{R}$	$\frac{\pi y}{R}=90°$: $\frac{w_1}{R}$
0	1.0000	1.0000	0.9660	0.8704	0.7286	0.5625	0.3962	0.25000
15°	0.9660	0.9829	0.9330	0.8390	0.6998	0.5373	0.3750	0.2333
30°	0.8704	0.9330	0.8390	0.7500	0.6187	0.4615	0.3163	0.1875
45°	0.7286	0.8536	0.6998	0.6187	0.50000	0.3643	0.2333	0.1250
60°	0.5625	0.7500	0.5373	0.4665	0.3643	0.2500	0.1440	0.0625
75°	0.3962	0.6295	0.3750	0.3163	0.2333	0.1440	0.0670	0.0167
90°	0.2500	0.50000	0.2333	0.1875	0.1250	0.0625	0.0167	0
105°	0.1373	0.3706	0.1250	0.0922	0.0503	0.0145	0	0.0167
120°	0.0625	0.2500	0.0543	0.0335	0.0107	0	0.0145	0.0625
135°	0.0215	0.1465	0.0168	0.0063	0	0.0107	0.0503	0.1250
150°	0.0045	0.0670	0.0025	0	0.0063	0.0335	0.0922	0.1875
165°	0.0003	0.0170	0	0.0025	0.0168	0.0543	0.1250	0.2338
180°	0	0	0.0003	0.0045	0.0215	0.0625	0.1373	0.2500

$$\frac{w_3}{R} = \frac{1}{4}\left[\cos\frac{2nx}{R} + \cos\frac{2ny}{R}\right], \quad \text{!!! Not Used !!!}$$

$\frac{nx}{R}$	$\frac{ny}{R}=0$ $\frac{w_3}{R}$	$\frac{ny}{R}=15^\circ$ $\frac{w_3}{R}$	$\frac{ny}{R}=30^\circ$ $\frac{w_3}{R}$	$\frac{ny}{R}=45^\circ$ $\frac{w_3}{R}$	$\frac{ny}{R}=60^\circ$ $\frac{w_3}{R}$	$\frac{ny}{R}=75^\circ$ $\frac{w_3}{R}$	$\frac{ny}{R}=90^\circ$ $\frac{w_3}{R}$	
0	0.50000	0.4665	0.37500	0.25000	0.1250	0.0335	0	
15°	0.4665	0.4330	0.3415	0.2165	0.0915	0	−0.0335	
30°	0.3750	0.3415	0.2500	0.1250	0	−0.0915	−0.1250	
45°	0.2500	0.2165	0.1250	0	−0.1250	−0.2165	−0.2500	
60°	0.1250	0.0915	0	−0.1250	−0.2500	−0.3415	−0.3750	
75°	0.0335	0	−0.0915	−0.2165	−0.3415	−0.4330	−0.4665	
90°	0	−0.0335	−0.1250	−0.2500	−0.3750	−0.4665	−0.5000	
105°	0.0335	0	−0.0915	−0.2165	−0.3415	−0.4330	−0.4665	
120°	0.1250	0.0915	0	−0.1250	−0.2500	−0.3415	−0.3750	
135°	0.2500	0.2165	0.1250	0	−0.1250	−0.2165	−0.2500	
150°	0.3750	0.3415	0.2500	0.1250	0	−0.0915	−0.1250	
165°	0.4665	0.4330	0.3415	0.2165	0.0915	0	−0.0335	
180°	0.5000	0.4660	0.3750	0.2500	0.1250	0.0335	0	

$$\frac{w}{R} = \left(f_0 + \frac{1}{4}f_1\right) + \left(\frac{1}{2}f_1 + f_2\right)\left[\cos\frac{mx}{R}\cos\frac{ny}{R} + \frac{1}{4}\cos\frac{2mx}{R} + \frac{1}{4}\cos\frac{2ny}{R}\right]$$

$$- f_2\cos\frac{mx}{R}\cos\frac{ny}{R}$$

$$= \left(f_0 - \frac{1}{2}f_2\right) + \left(f_1 + 2f_2\right)\cos^2\frac{(mx+ny)}{2R}\cos^2\frac{(mx-ny)}{2R}$$

$$- f_2\cos\frac{mx}{R}\cos\frac{ny}{R}$$

Amplitude ratio $\quad \frac{w_1}{w_2} = \frac{f_1 + 2f_2}{-f_2} = -\frac{1+2\rho}{\rho}$

$w_1 : w_2 = 1 : -\rho/1+2\rho$

$\mu = 1.000$; $n = 26$, $\xi = 0.5$

$\rho = -0.2667875$; $w_1 : w_2 = 1 : 0.57198$

$\frac{nx}{R}$	ny/R						
	0	15°	30°	45°	60°	75°	90°
0	1.5720	1.5185	1.3657	1.1330	0.8485	0.5442	0.25000
15°	1.5185	1.4667	1.3175	1.0905	0.8136	0.5180	0.2333
30°	1.3657	1.3175	1.1790	0.9689	0.7142	0.4447	0.1875
45°	1.1330	1.0905	0.9689	0.7860	0.5666	0.3380	0.1250
60°	0.8485	0.8136	0.7142	0.5666	0.3930	0.2180	0.0625
75°	0.5442	0.5180	0.4447	0.3380	0.2180	0.1053	0.0167
90°	0.2500	0.2330	0.1875	0.1250	0.0625	0.0167	0
105°	−0.0107	−0.0180	−0.0360	−0.0544	−0.0595	−0.0383	0.0167
120°	−0.2235	−0.2220	−0.2142	−0.1916	−0.1430	−0.0595	0.0625
135°	−0.3829	−0.3739	−0.3437	−0.2860	−0.1916	−0.0544	0.1250
150°	−0.4908	−0.4760	−0.4290	−0.3439	−0.2229	−0.0360	0.1875
165°	−0.5522	−0.5337	−0.4760	−0.3738	−0.2397	−0.0180	0.2333
180°	−0.5720	−0.5522	−0.4908	−0.3829	−0.2235	−0.0107	0.2500

$\mu = 1.000$; $n = 26$, $\xi = 2.000$ 787

$\rho = -0.1467044$; $w_1 : w_2 = 1.0000 : 0.20762$

$\frac{\pi x}{R}$	$\pi y/R$						
	0	15°	30°	45°	60°	75°	90°
0°	1.20762	1.1665	1.0502	0.8754	0.6663	0.4499	0.2500
15°	1.1665	1.1267	1.0127	0.8416	0.6376	0.4269	0.2333
30°	1.0502	1.0127	0.9057	0.7458	0.5564	0.3628	0.1875
45°	0.8754	0.8416	0.7458	0.1038	0.4377	0.2713	0.1250
60°	0.6663	0.6376	0.5564	0.4377	0.3019	0.1709	0.0625
75°	0.4499	0.4269	0.3628	0.2713	0.1709	0.0809	0.0167
90°	0.2500	0.2333	0.1875	0.1250	0.0625	0.0167	0
105°	0.0836	0.0731	0.0456	0.0123	−0.0124	−0.0139	+0.0167
120°	−0.0413	−0.0460	−0.0564	−0.0627	−0.0519	−0.0124	+0.0625
135°	−0.1253	−0.1250	−0.1208	−0.1038	−0.0627	+0.0123	+0.1250
150°	−0.1753	−0.1712	−0.1557	−0.1208	−0.0564	+0.0457	+0.1875
165°	−0.2002	−0.1937	−0.1712	−0.1250	−0.0460	+0.0731	+0.2333
180°	−0.20762	−0.2002	−0.1753	−0.1253	−0.0413	+0.0836	+0.2500

$\mu = 1.000$; $n = 90$; $\xi = 4.00$

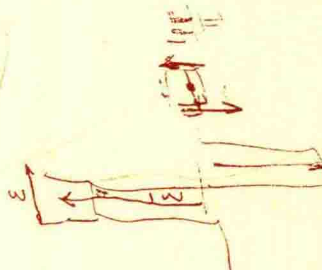
$\rho = -0.1005343$; $w_1 : w_2 = 1 : 0.12584$

$\frac{nx}{R}$	ny/R						
	0	15°	30°	45°	60°	75°	90°
0°	1.1258	1.0875	0.9794	0.8176	0.6254	0.4288	0.2500
15°	1.0875	1.0504	0.9443	0.7857	0.5981	0.4065	0.2333
30°	0.9794	0.9443	0.8444	0.6958	0.5210	0.3445	0.1875
45°	0.8176	0.7857	0.6958	0.5629	0.4088	0.2563	0.1250
60°	0.6254	0.5981	0.5210	0.4088	0.2815	0.1603	0.0625
75°	0.4288	0.4065	0.3445	0.2563	0.1603	0.0754	0.0167
90°	0.2500	0.2333	0.1875	0.1250	0.0625	0.0167	0
105°	0.1047	0.0935	0.0640	0.0273	−0.0017	−0.0084	0.167
120°	−0.0004	−0.0065	−0.0209	−0.0351	−0.0315	−0.0017	0.0625
135°	−0.0675	−0.0691	−0.0708	−0.0629	−0.0351	0.0273	0.1250
150°	−0.1044	−0.1028	−0.0944	−0.0708	−0.0209	0.0640	0.1875
165°	−0.1212	−0.1174	−0.1028	−0.0691	−0.0065	0.0935	0.2333
180°	−0.1258	−0.1212	−0.1044	−0.0675	−0.0004	0.1047	0.2500

$\mu = 1.000; \quad n = 10; \quad \xi = 16.21518$

$\rho = -0.0749409; \qquad w_1 : w_2 = 1 : 0.08815$

$\frac{nx}{R}$	ny/R						
	0	15°	30°	45°	60°	75°	90°
0	1.0882	1.0511	0.9467	0.7909	0.6066	0.4140	0.2500
15°	1.0511	1.0152	0.9127	0.7600	0.5799	0.3970	0.2333
30°	0.9467	0.9127	0.8161	0.6727	0.5047	0.3361	0.1875
45°	0.7909	0.7600	0.6727	0.5441	0.3955	0.2494	0.1250
60°	0.6066	0.5799	0.5047	0.3955	0.2720	0.1554	0.0625
75°	0.4140	0.3970	0.3361	0.2494	0.1554	0.0729	0.0167
90°	0.2500	0.2333	0.1875	0.1250	0.0625	0.0167	0
105°	0.1145	0.1030	0.0724	0.0342	0.0031	−0.0059	0.0167
120°	0.0184	0.0117	−0.0047	−0.0205	−0.0220	0.0031	0.0625
135°	−0.0408	−0.0434	−0.0477	−0.0441	−0.0205	0.0342	0.1250
150°	−0.0718	−0.0712	−0.0661	−0.0477	−0.0047	0.0724	0.1875
165°	−0.0848	−0.0822	−0.0712	−0.0434	0.0117	0.1030	0.2333
180°	−0.0882	−0.0848	−0.0718	−0.0408	0.0184	0.1145	0.2500



$\mu = 2.$; $n = 23$; $\xi = 2.5$

$\rho = -0.2996830$; $w_1 : w_2 = 1 : 0.74802$

$\frac{nx}{R}$	ny/R						
	0	15°	30°	45°	60°	75°	90°
0	1.74802	1.6885	1.5182	1.2575	0.9365	0.5838	0.2500
15°	1.6885	1.6309	1.4647	1.2107	0.8986	0.5620	0.2333
30°	1.5182	1.4647	1.3110	1.0767	0.7904	0.4839	0.1875
45°	1.2575	1.2107	1.0767	0.8740	0.6288	0.3702	0.1250
60°	0.9365	0.8986	0.7904	0.6288	0.4370	0.2408	0.0625
75°	0.5838	0.5620	0.4839	0.3702	0.2408	0.1171	0.0167
90°	0.2500	0.2333	0.1875	0.1250	0.0625	0.0167	0
105°	-0.0563	-0.0620	-0.0754	-0.0866	-0.0823	-0.0501	0.0167
120°	-0.3115	-0.3070	-0.2904	-0.2538	-0.1870	-0.0823	0.0625
135°	-0.5074	-0.4941	-0.4517	-0.3740	-0.2538	-0.0866	0.1250
150°	-0.6433	-0.6232	-0.5610	-0.4517	-0.2904	-0.0754	0.1875
165°	-0.7222	-0.6979	-0.6232	-0.4941	-0.3070	-0.0620	0.2333
180°	-0.7480	-0.7222	-0.6433	-0.5074	-0.3115	-0.0563	0.2500

A1

$\mu = 2,\ n = 17;\quad \xi = 5.5$

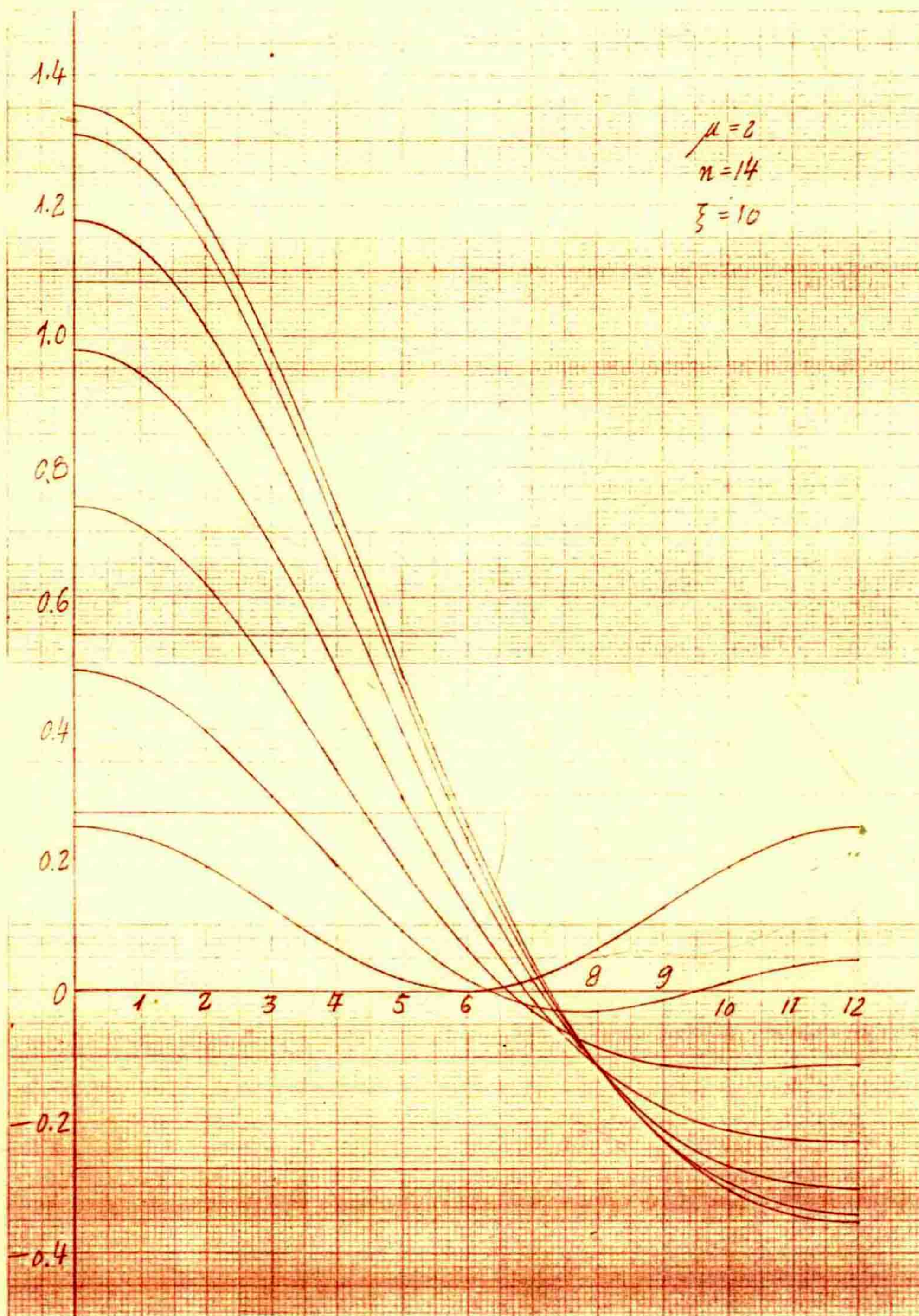
$\rho = -0.2447866;\quad w_1 : w_2 = 1 : 0.47957$

$\frac{nx}{R}$	ny/R						
	0°	15°	30°	45°	60°	75°	90°
0°	1.4796	1.4292	1.2857	1.0677	0.8023	0.5203	0.2500
15°	1.4292	1.3804	1.2402	1.0273	0.7689	0.4949	0.2333
30°	1.2857	1.2402	1.1097	0.9123	0.6742	0.4238	0.1875
45°	1.0677	1.0273	0.9123	0.7398	0.5339	0.3211	0.1250
60°	0.8023	0.7689	0.6742	0.5339	0.3699	0.2061	0.0625
75°	0.5203	0.4949	0.4238	0.3211	0.2061	0.0991	0.0167
90°	0.2500	0.2333	0.1875	0.1250	0.0625	0.0167	0
105°	0.0132	0.0051	-0.0153	-0.0375	-0.0476	-0.0321	0.0167
120°	-0.1773	-0.1773	-0.1742	-0.1589	-0.1199	-0.0476	0.0625
135°	-0.3176	-0.3107	-0.2873	-0.2398	-0.1589	-0.0375	0.1250
150°	-0.4108	-0.3987	-0.3597	-0.2873	-0.1742	-0.0153	0.1875
165°	-0.4629	-0.4474	-0.3987	-0.3107	-0.1773	0.0051	0.2333
180°	-0.4796	-0.4629	-0.4108	-0.3176	-0.1773	0.0132	0.2500

$\mu = 2, \quad n = 14; \quad \xi = 10$

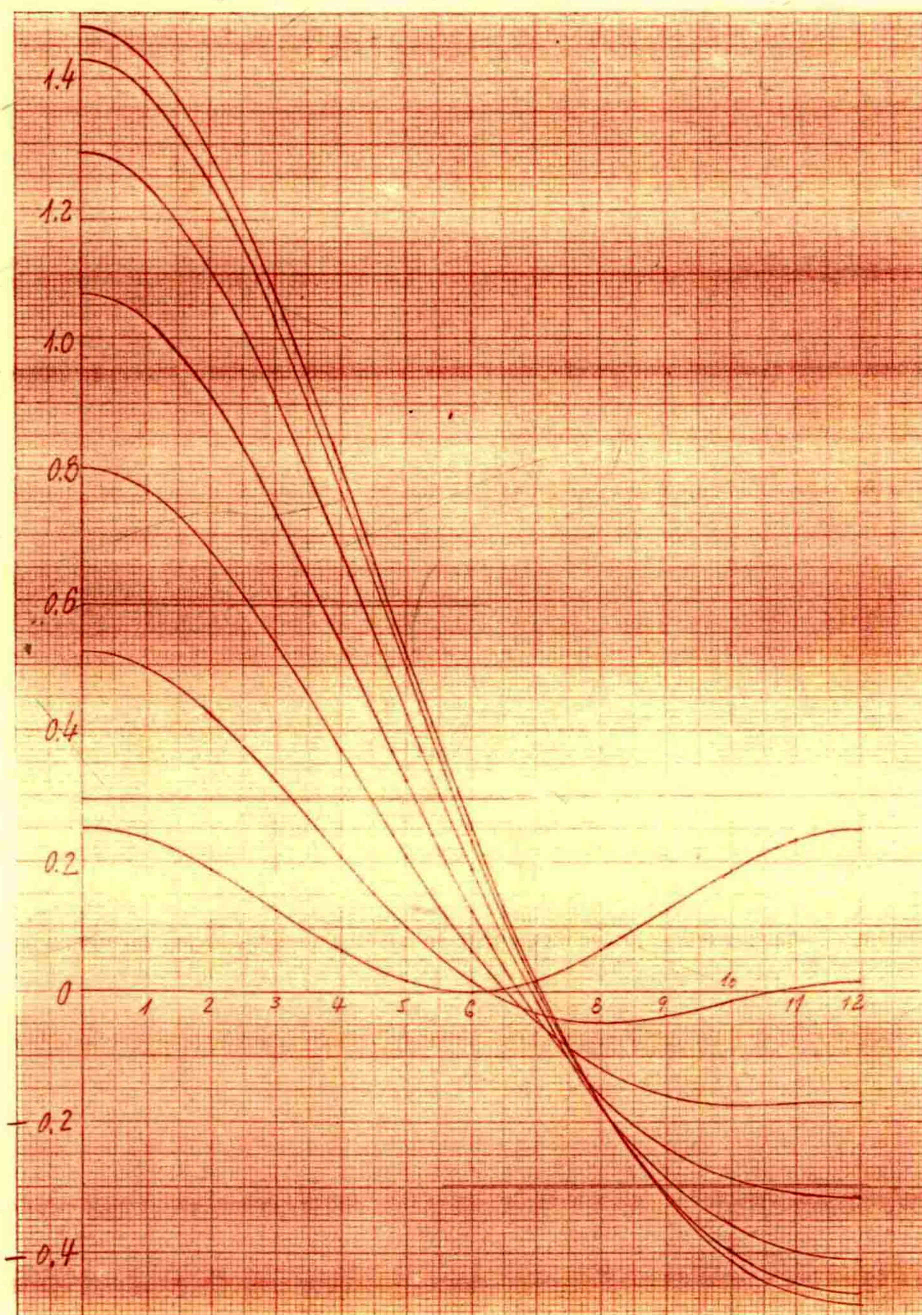
$\rho = -0.2071031; \quad w_1 : w_2 = 1 : 0.35354$

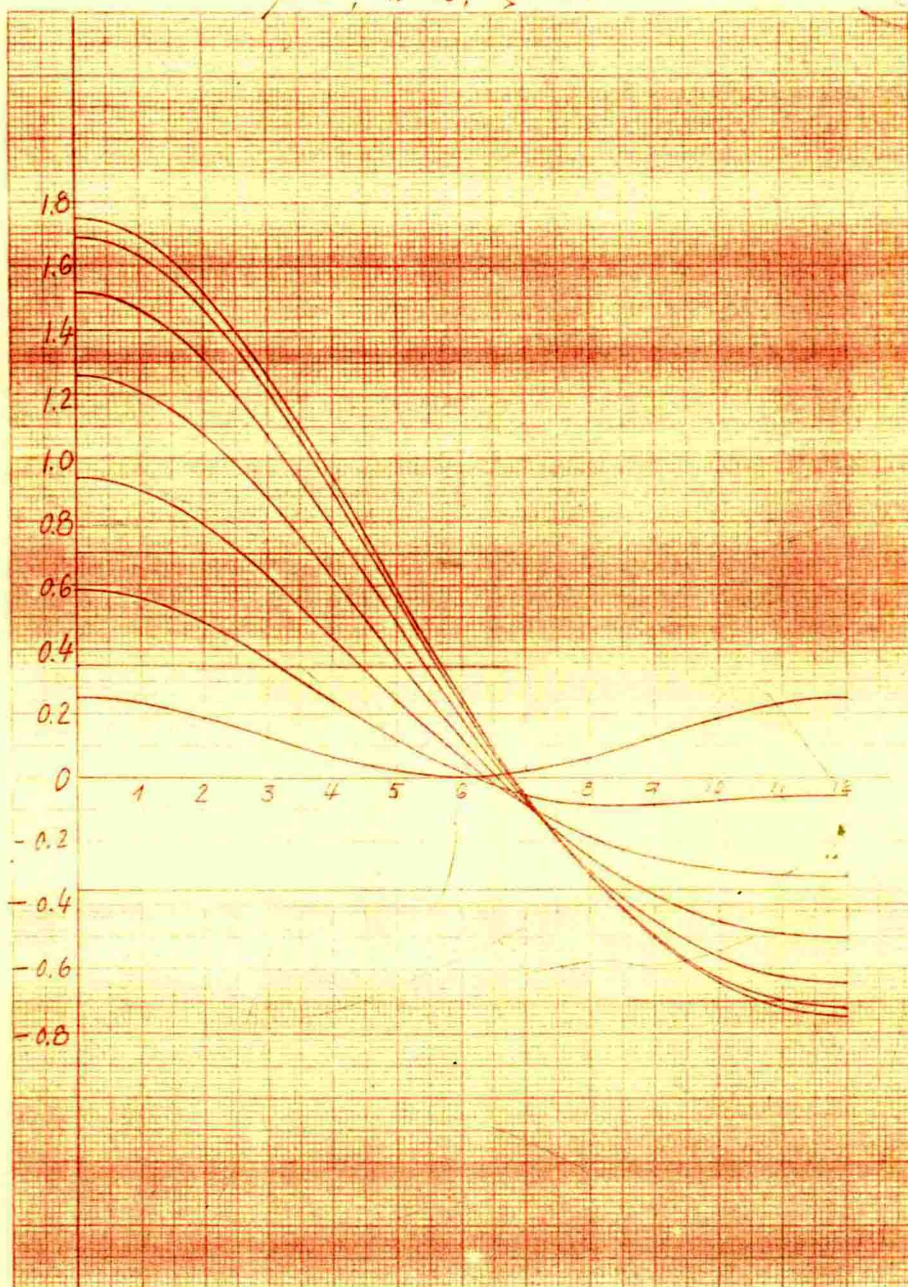
$\frac{\pi x}{R}$	$\pi y / R$						
	0°	15°	30°	45°	60°	75°	90°
0°	1.3535	1.3075	1.1766	0.9786	0.7393	0.4877	0.2500
15°	1.3075	1.2629	1.1347	0.9413	0.7081	0.4634	0.2333
30°	1.1766	1.1347	1.0152	0.8352	0.6196	0.3955	0.1875
45°	0.9786	0.9413	0.8352	0.6768	0.4893	0.2980	0.1250
60°	0.7393	0.7081	0.6196	0.4893	0.3384	0.1897	0.0625
75°	0.4877	0.4634	0.3955	0.2980	0.1897	0.0907	0.0167
90°	0.2500	0.2333	0.1875	0.1250	0.0625	0.0167	0
105°	0.0458	0.0366	0.0130	-0.0144	-0.0312	-0.0237	0.0167
120°	-0.1143	-0.1165	-0.1196	-0.1143	-0.0884	-0.0312	0.0625
135°	-0.2285	-0.2247	-0.2102	-0.1768	-0.1143	-0.0144	0.1250
150°	-0.3017	-0.2932	-0.2652	-0.2102	-0.1196	0.0130	0.1875
165°	-0.3412	-0.3299	-0.2932	-0.2247	-0.1165	0.0366	0.2333
180°	-0.3535	-0.3412	-0.3017	-0.2285	-0.1143	0.0458	0.2500



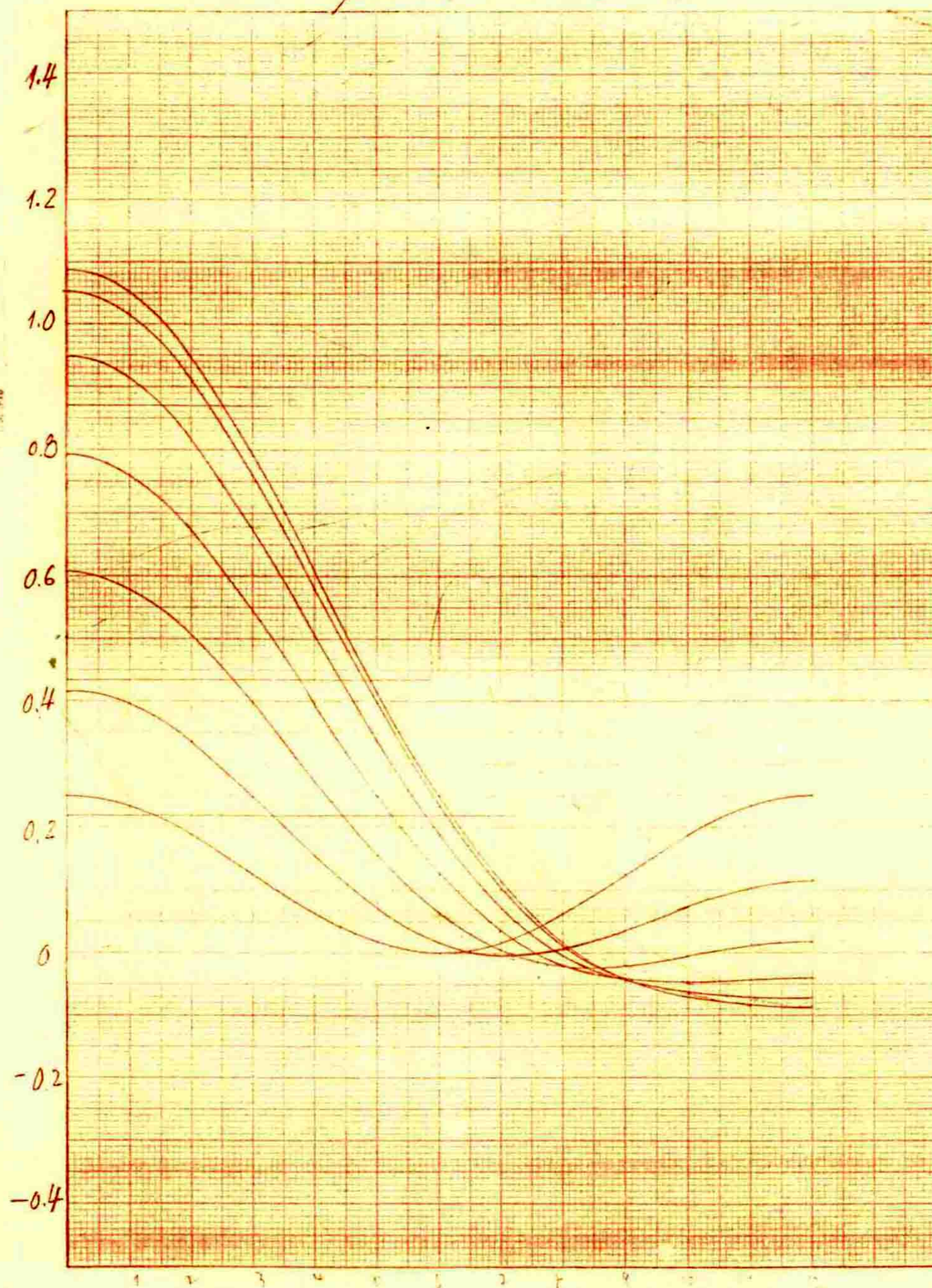
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
−0.2
−0.4
1
2
3
4
5
6
8
9
10
11
12
$\mu = 2$
$n = 14$
$\xi = 10$

$\mu = 2.0$; $n = 17$, $\xi = 5.5$





$\mu = 2, \quad n = 23, \quad \xi = 2.5$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
1
2
3
4
5
6
8
9
10
11
12



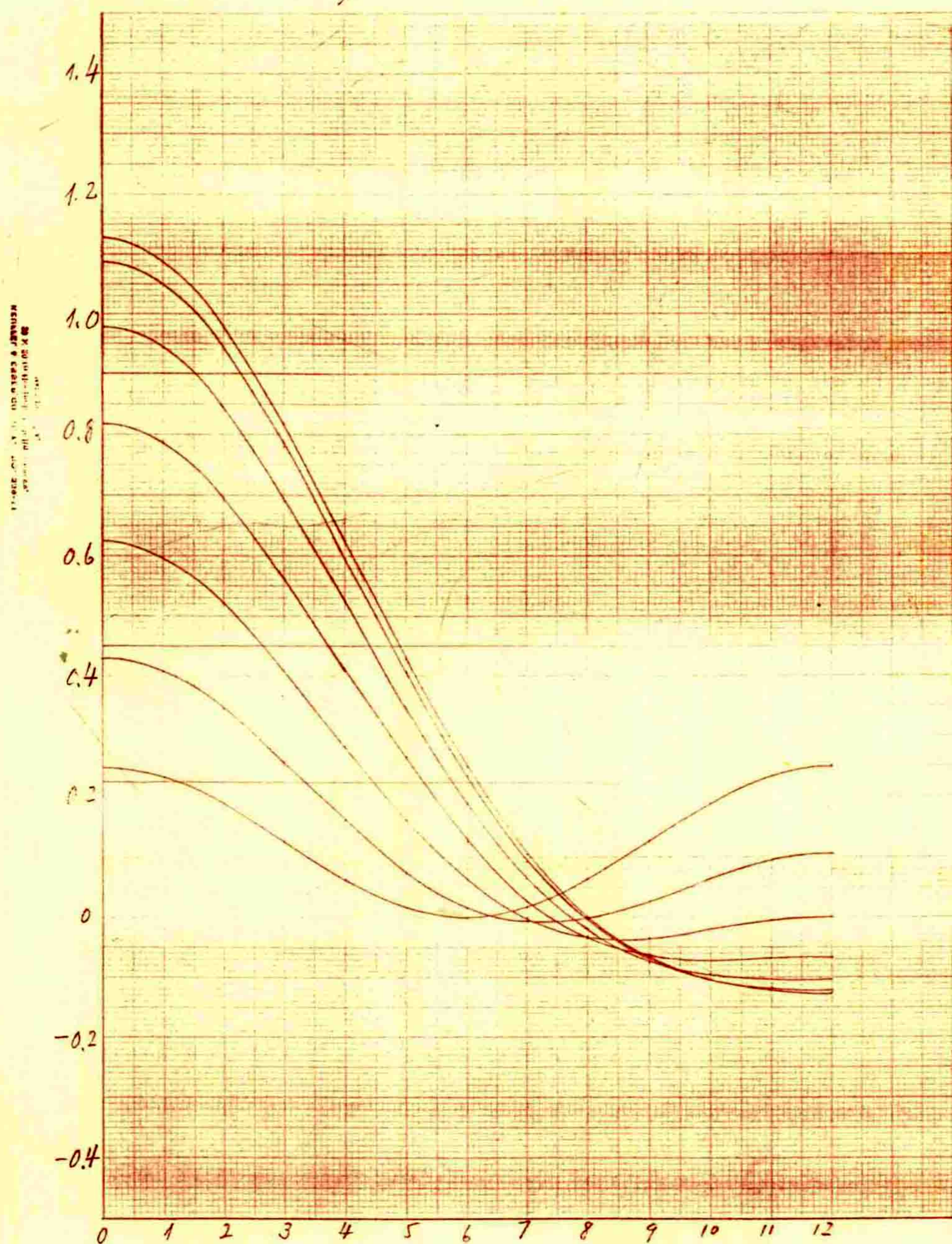
μ=1.000 ; n=10 ; ξ = 16.21518
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4

$\mu = 1.000; \; n = 26; \; \xi = 2.00$

1.4
1.2
1.0
0.8
0.6
0.4
0
−0.2
−0.4

0 1 2 3 4 5 6 7 8 9 10 11 12




$\mu = 1.000$; $n = 20$; $\xi = 4.0$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
1
2
3
4
5
6
7
8
9
10
11
12

$\mu = 1.00$; $\eta = 26$; $\xi = 0.5$

1.4

1.2

1.0

.8

.6

.4

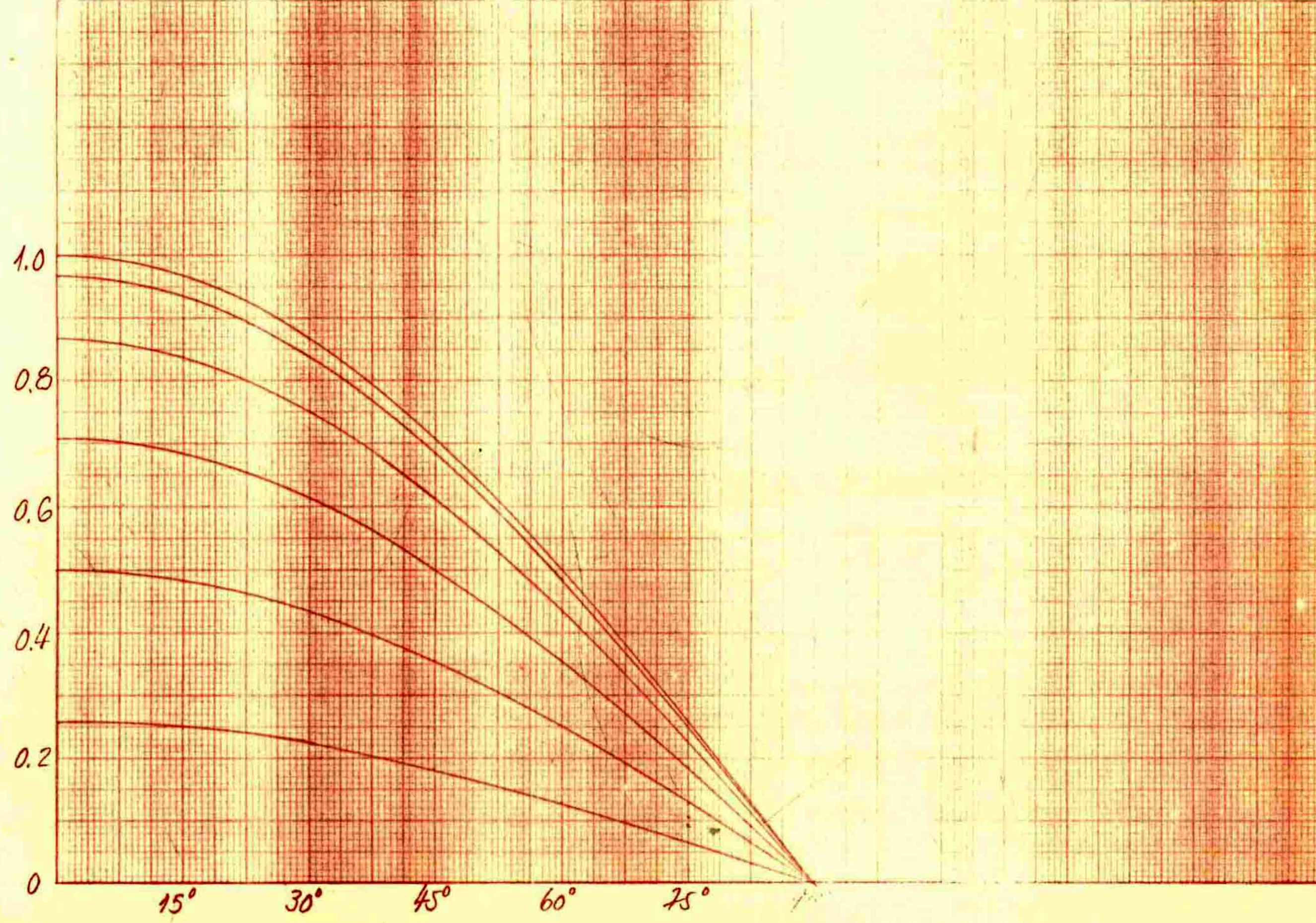
.2

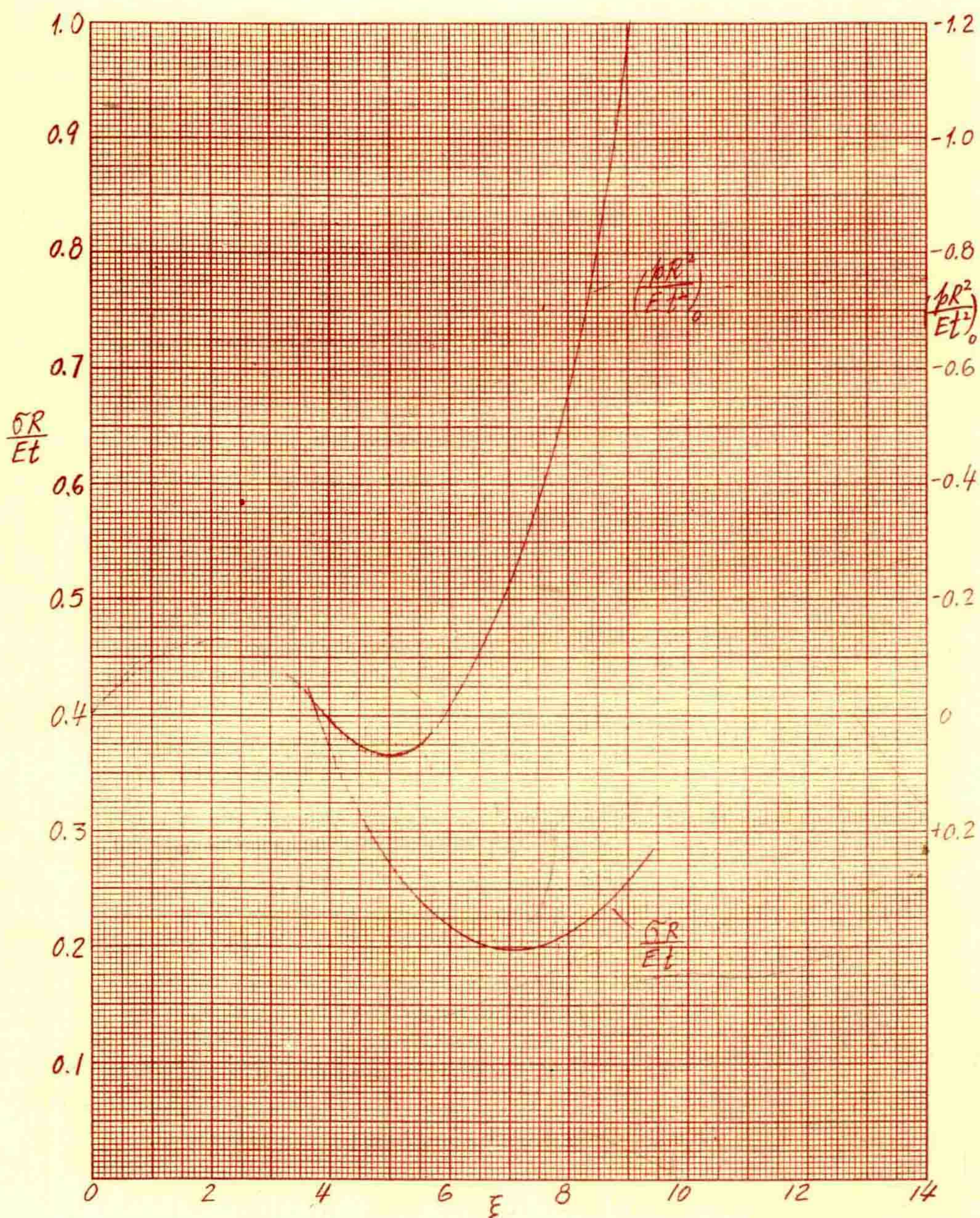
0

-.2

-.4

1 2 3 4 5 6 7 8 9 10 11 12


$\mu = 1.00$; $\varkappa = 26$; $\xi = 0$
1.0
0.8
0.6
0.4
0.2
0
15°
30°
45°
60°
75°


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$\frac{\sigma R}{Et}$
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
0
+0.2
$\left(\frac{pR^2}{Et^2}\right)_0$
0
2
4
6
8
10
12
14
ξ

$$+(\sigma_x+\sigma)\frac{\partial^2 w}{\partial x^2} \quad ; \quad +\sigma_y\frac{\partial^2 w}{\partial y^2} \quad ; \quad +2\tau_{xy}\frac{\partial^2 w}{\partial x \partial y}$$ <u>a</u>

these are the forces which will help the buckling

$$+(\sigma_x+\sigma) = +E\mu^2\Big[\frac{B}{4}\cos\frac{2ny}{R} + \frac{C}{(1+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{ny}{R} + \frac{D}{(1+9\mu^2)^2}\cos\frac{3mx}{R}\cos\frac{ny}{R}$$

$$+\frac{9G}{(9+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{3ny}{R} + \frac{H}{4(1+\mu^2)^2}\cos\frac{2mx}{R}\cos\frac{2ny}{R}\Big]$$

$$+\sigma_y = +E\mu^2\Big[\frac{A}{4\mu^2}\cos\frac{2mx}{R} + \frac{\mu^2 C}{(1+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{ny}{R}$$

$$+\frac{9\mu^2 D}{(1+9\mu^2)^2}\cos\frac{3mx}{R}\cos\frac{ny}{R} + \frac{\mu^2 G}{(9+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{3ny}{R} + \frac{\mu^2 H}{4(1+\mu^2)^2}\cos\frac{2mx}{R}\cos\frac{2ny}{R}\Big]$$

$$+\tau_{xy} = +E\mu^2\Big[\frac{\mu C}{(1+\mu^2)^2}\sin\frac{mx}{R}\sin\frac{ny}{R} + \frac{3\mu D}{(1+9\mu^2)^2}\sin\frac{3mx}{R}\sin\frac{ny}{R} +$$

$$+\frac{3\mu G}{(9+\mu^2)^2}\sin\frac{mx}{R}\sin\frac{3ny}{R} + \frac{\mu H}{4(1+\mu^2)^2}\sin\frac{2mx}{R}\sin\frac{2ny}{R}\Big]$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{n^2}{R^2}\mu^2\Big\{\frac{1}{2}f_1\Big[\cos\frac{mx}{R}\cos\frac{ny}{R} + \cos\frac{2mx}{R}\Big] + f_2\cos\frac{2mx}{R}\Big\}$$

$$= -\frac{n^2}{R}\mu^2\Big\{\frac{1}{2}f_1\cos\frac{mx}{R}\cos\frac{ny}{R} + \Big(\frac{1}{2}f_1+f_2\Big)\cos\frac{2mx}{R}\Big\}$$

$$\frac{\partial^2 w}{\partial y^2} = -\frac{n^2}{R}\Big\{\frac{1}{2}f_1\cos\frac{mx}{R}\cos\frac{ny}{R} + \Big(\frac{1}{2}f_1+f_2\Big)\cos\frac{2ny}{R}\Big\}$$

$$\frac{\partial^2 w}{\partial x\partial y} = \frac{n^2}{R}\mu\Big[\frac{1}{2}f_1\sin\frac{mx}{R}\sin\frac{ny}{R}\Big\}$$

$$+(\sigma_x+\sigma)\frac{\partial^2 w}{\partial x^2} = -\frac{n^2}{R^2}E\mu^2\Big[\mu^2\frac{Bf_1}{8}\cos\frac{mx}{R}\cos\frac{ny}{R}\cos\frac{2ny}{R} + \frac{Cf_1\mu^2}{2(1+\mu^2)^2}\cos^2\frac{mx}{R}\cos^2\frac{ny}{R} + \frac{Df_1\mu^2}{2(1+9\mu^2)^2}\cos\frac{mx}{R}\cos\frac{3mx}{R}\cos^2\frac{ny}{R}$$

$$+\frac{9Gf_1\mu^2}{2(9+\mu^2)^2}\cos^2\frac{mx}{R}\cos\frac{ny}{R}\cos\frac{3ny}{R} + \frac{Hf_1\mu^2}{8(1+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{ny}{R}\cos\frac{2ny}{R} +$$

$$+\mu^2\left(\frac{1}{2}f_1+f_2\right)\Big\{\frac{B}{4}\cos\frac{2mx}{R}\cos\frac{2ny}{R} + \frac{C}{(1+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{ny}{R} + \frac{D}{(1+9\mu^2)^2}\cos\frac{2mx}{R}\cos\frac{3mx}{R}\cos\frac{ny}{R}$$

$$+\frac{9G}{(9+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{mx}{R}\cos\frac{3ny}{R} + \frac{H}{4(1+\mu^2)^2}\cos^2\frac{2mx}{R}\cos\frac{2ny}{R}\Big\}\Big]$$

$$+\sigma_y\frac{\partial^2 w}{\partial y^2} = -\frac{n^2}{R^2}E\mu^2\Big[\frac{Af_1\mu^2}{8\mu^4}\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{ny}{R} + \frac{Cf_1\mu^2}{2(1+\mu^2)^2}\cos^2\frac{mx}{R}\cos^2\frac{ny}{R} + \frac{9Df_1\mu^2}{2(1+9\mu^2)^2}\cos\frac{mx}{R}\cos\frac{3mx}{R}\cos^2\frac{ny}{R}$$

$$+\frac{Gf_1\mu^2}{2(9+\mu^2)^2}\cos^2\frac{mx}{R}\cos\frac{ny}{R}\cos\frac{3ny}{R} + \frac{Hf_1\mu^2}{8(1+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{ny}{R}\cos\frac{2ny}{R} +$$

$$+\mu^2\left(\frac{1}{2}f_1+f_2\right)\Big\{\frac{A}{4\mu^4}\cos\frac{2mx}{R}\cos\frac{2ny}{R} + \frac{C}{(1+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{ny}{R}\cos\frac{2ny}{R} + \frac{9D}{(1+9\mu^2)^2}\cos\frac{3mx}{R}\cos\frac{ny}{R}\cos\frac{2ny}{R}$$

$$+\frac{G}{(9+\mu^2)^2}\cos\frac{mx}{R}\cos\frac{2ny}{R}\cos\frac{3ny}{R} + \frac{H}{4(1+\mu^2)^2}\cos\frac{2mx}{R}\cos^2\frac{2ny}{R}\Big\}\Big]$$

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$$+2T_{xy}\frac{\partial^2 w}{\partial x \partial y} = \frac{n^2}{R^2}E\mu^2\left[-\frac{2\mu^2 Cf_1}{2(1+\mu^2)}\sin^2\frac{mx}{R}\sin^2\frac{ny}{R} - \frac{6\mu^2 Df_1}{2(1+9\mu^2)^2}\sin\frac{mx}{R}\sin\frac{3mx}{R}\sin^2\frac{ny}{R}\right.$$

$$\left. - \frac{6\mu^2 Gf_1}{2(9+\mu^2)^2}\sin^2\frac{mx}{R}\sin\frac{ny}{R}\sin\frac{3ny}{R} - \frac{9\mu^2 Hf_1}{8(1+\mu^2)^2}\sin\frac{mx}{R}\sin\frac{2mx}{R}\sin\frac{ny}{R}\sin\frac{2ny}{R}\right]$$

$$p = -\frac{n^2}{R^2}E\mu^4\left[\frac{Bf_1}{8}\cos\frac{mx}{R}\cos\frac{ny}{R}\cos\frac{2ny}{R} + \frac{Af_1}{8\mu^4}\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{ny}{R} + \frac{Cf_1}{(1+\mu^2)^2}\left(\cos^2\frac{mx}{R}\cos^2\frac{ny}{R} - \sin^2\frac{mx}{R}\sin^2\frac{ny}{R}\right)\right.$$

$$+ \frac{Df_1}{(1+9\mu^2)^2}\left(5\cos\frac{mx}{R}\cos\frac{3mx}{R}\cos^2\frac{ny}{R} - 3\sin\frac{mx}{R}\sin\frac{3mx}{R}\sin^2\frac{ny}{R}\right)$$

$$+ \frac{Gf_1}{(9+\mu^2)^2}\left(15\cos^2\frac{mx}{R}\cos\frac{ny}{R}\cos\frac{3ny}{R} - 3\sin^2\frac{mx}{R}\sin\frac{ny}{R}\sin\frac{3ny}{R}\right)$$

$$+ \frac{Hf_1}{4(1+\mu^2)^2}\left(\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{ny}{R}\cos\frac{2ny}{R} - \sin\frac{mx}{R}\sin\frac{2mx}{R}\sin\frac{ny}{R}\sin\frac{2ny}{R}\right)$$

$$+ \left(\frac{1}{2}f_1 + f_2\right)\left\{\frac{1}{4}\left(\frac{A}{\mu^4}+B\right)\cos\frac{2mx}{R}\cos\frac{2ny}{R} + \frac{C}{(1+\mu^2)^2}\left(\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{ny}{R} + \cos\frac{mx}{R}\cos\frac{ny}{R}\cos\frac{2ny}{R}\right)\right.$$

$$+ \frac{D}{(1+9\mu^2)^2}\left\{\cos\frac{2mx}{R}\cos\frac{3mx}{R}\cos\frac{ny}{R} + 9\cos\frac{3mx}{R}\cos\frac{ny}{R}\cos\frac{2ny}{R}\right\} + \frac{G}{(9+\mu^2)^2}\left\{9\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{3ny}{R} + \cos\frac{mx}{R}\cos\frac{2ny}{R}\cos\frac{3ny}{R}\right\}$$

$$\left.\left. + \frac{H}{4(1+\mu^2)^2}\left\{\cos^2\frac{2mx}{R}\cos\frac{2ny}{R} + \cos\frac{2mx}{R}\cos^2\frac{2ny}{R}\right\}\right\}\right]$$

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$$\delta' = \frac{t n^2}{R} E \mu^4 f_1 \left[\frac{B}{16} \cos\frac{mx}{R} \left(\cos\frac{ny}{R} + \cos\frac{3ny}{R} \right) + \frac{A}{16\mu^4} \left(\cos\frac{mx}{R} + \cos\frac{3mx}{R} \right) \cos\frac{ny}{R} + \frac{C}{2(1+\mu^2)^2} \left(\cos\frac{2mx}{R} + \cos\frac{2ny}{R} \right) \right.$$

$$+ \frac{D}{(1+9\mu^2)^2} \left\{ \frac{1}{2} \left(\cos\frac{2mx}{R} + \cos\frac{4mx}{R} \cos\frac{2ny}{R} \right) + 2 \left(\cos\frac{4mx}{R} + \cos\frac{2mx}{R} \cos\frac{2ny}{R} \right) \right\}$$

$$+ \frac{G}{(9+\mu^2)^2} \left\{ \frac{1}{2} \left(\cos\frac{2ny}{R} + \cos\frac{2mx}{R} \cos\frac{4ny}{R} \right) + 2 \left(\cos\frac{4ny}{R} + \cos\frac{2mx}{R} \cos\frac{2ny}{R} \right) \right\}$$

$$+ \frac{H}{8(1+\mu^2)^2} \left\{ \cos\frac{3mx}{R} \cos\frac{ny}{R} + \cos\frac{mx}{R} \cos\frac{3ny}{R} \right\} + \left(\frac{1}{2} + \rho \right) \left\{ \frac{1}{4} \left(\frac{A}{\mu^4} + B \right) \cos\frac{2mx}{R} \cos\frac{2ny}{R} + \right.$$

$$+ \frac{C}{2(1+\mu^2)^2} \left[\left(\cos\frac{mx}{R} + \cos\frac{3mx}{R} \right) \cos\frac{ny}{R} + \cos\frac{mx}{R} \left(\cos\frac{ny}{R} + \cos\frac{3ny}{R} \right) \right]$$

$$+ \frac{D}{2(1+9\mu^2)^2} \left[\left(\cos\frac{mx}{R} + \cos\frac{5mx}{R} \right) \cos\frac{ny}{R} + 9 \cos\frac{3mx}{R} \left(\cos\frac{ny}{R} + \cos\frac{3ny}{R} \right) \right]$$

$$+ \frac{G}{2(9+\mu^2)^2} \left[9 \left(\cos\frac{mx}{R} + \cos\frac{3mx}{R} \right) \cos\frac{3ny}{R} + \cos\frac{mx}{R} \left(\cos\frac{ny}{R} + \cos\frac{5ny}{R} \right) \right]$$

$$\left. \left. + \frac{H}{8(1+\mu^2)^2} \left[\left(1 + \cos\frac{4mx}{R} \right) \cos\frac{2ny}{R} + \cos\frac{2mx}{R} \left(1 + \cos\frac{4ny}{R} \right) \right] \right\} \right] + \frac{t}{R} E \mu^4 \left[\frac{A}{4\mu^4} \cos\frac{2mx}{R} + \right.$$

$$\left. + \frac{C}{(1+\mu^2)^2} \cos\frac{mx}{R} \cos\frac{ny}{R} + \frac{9D}{(1+9\mu^2)^2} \cos\frac{3mx}{R} \cos\frac{ny}{R} + \frac{G}{(9+\mu^2)^2} \cos\frac{mx}{R} \cos\frac{3ny}{R} + \frac{H}{4(1+\mu^2)^2} \cos\frac{2mx}{R} \cos\frac{2ny}{R} \right] \rho$$

$$\frac{pR}{tE}\frac{1}{\mu^4} = \cos\frac{2mx}{R}\left\{-\eta\xi\left(\frac{C^{\vee}}{2(1+\mu^2)^2} + \frac{1}{2}\frac{D}{(1+9\mu^2)^2} + \frac{H}{8(1+\mu^2)^2}\left(\frac{1}{2}+\rho\right)\right) + \frac{A}{4\mu^4}\right\} \quad \underline{e}$$

$$+ \cos\frac{2ny}{R}\left\{-\eta\xi\left(\frac{C^{\vee}}{2(1+\mu^2)^2} + \frac{1}{2}\frac{G}{(9+\mu^2)^2} + \frac{H}{8(1+\mu^2)^2}\left(\frac{1}{2}+\rho\right)\right)\right\}$$

$$+ \cos\frac{4mx}{R}\left\{-\eta\xi\left(\frac{2D}{(1+9\mu^2)^2}\right)\right\} + \cos\frac{4ny}{R}\left\{-\eta\xi\left(\frac{2G}{(9+\mu^2)^2}\right)\right\}$$

$$+ \cos\frac{mx}{R}\cos\frac{ny}{R}\left\{-\eta\xi\left(\frac{B}{16} + \frac{A}{16\mu^4} + \left(\frac{1}{2}+\rho\right)\left\{\frac{C^{\vee}}{(1+\mu^2)^2} + \frac{D}{2(1+9\mu^2)^2} + \frac{G}{2(9+\mu^2)^2}\right\}\right) + \frac{C}{(1+\mu^2)^2}\right\}$$

$$+ \cos\frac{mx}{R}\cos\frac{3ny}{R}\left\{-\eta\xi\left(\frac{B}{16} + \frac{H}{8(1+\mu^2)^2} + \left(\frac{1}{2}+\rho\right)\left[\frac{C^{\vee}}{2(1+\mu^2)^2} + \frac{9G}{2(9+\mu^2)^2}\right]\right) + \frac{G}{(9+\mu^2)^2}\right\}$$

$$+ \cos\frac{3mx}{R}\cos\frac{ny}{R}\left\{-\eta\xi\left(\frac{A}{16\mu^4} + \frac{H}{8(1+\mu^2)^2} + \left(\frac{1}{2}+\rho\right)\left[\frac{C^{\vee}}{2(1+\mu^2)^2} + \frac{9D}{2(1+9\mu^2)^2}\right]\right) + \frac{9D}{(1+9\mu^2)^2}\right\}$$

$$+ \cos\frac{2mx}{R}\cos\frac{2ny}{R}\left\{-\eta\xi\left(\frac{2D}{(1+9\mu^2)^2} + \frac{2G}{(9+\mu^2)^2} + \left(\frac{1}{2}+\rho\right)\frac{1}{4}\left(\frac{A}{\mu^4}+B\right)\right) + \frac{H}{4(1+\mu^2)^2}\right\}$$

$$+ \cos\frac{mx}{R}\cos\frac{5ny}{R}\left\{-\left(\frac{1}{2}+\rho\right)\eta\xi\,\frac{G}{2(9+\mu^2)^2}\right\} + \cos\frac{5mx}{R}\cos\frac{ny}{R}\left\{-\left(\frac{1}{2}+\rho\right)\eta\xi\,\frac{D}{2(1+9\mu^2)^2}\right\}$$

$$- \cos\frac{2mx}{R}\cos\frac{4ny}{R}\left\{\frac{G}{2(9+\mu^2)^2} + \left(\frac{1}{2}+\rho\right)\frac{H}{8(1+\mu^2)^2}\right\}\eta\xi - \eta\xi\cos\frac{4mx}{R}\cos\frac{2ny}{R}\left\{\frac{D}{2(9+\mu^2)^2} + \left(\frac{1}{2}+\rho\right)\frac{H}{8(1+\mu^2)^2}\right\}$$

$$- \cos\frac{3mx}{R}\cos\frac{3ny}{R}\left\{\frac{9D}{2(1+9\mu^2)^2} + \frac{9G}{2(9+\mu^2)^2}\right\}\left(\frac{1}{2}+\rho\right)\eta\xi$$

$$\left(\frac{pR}{tE}\frac{1}{\mu^4}\right)_0 = -\eta\xi\left[\frac{B}{8} + \frac{A}{8\mu^4} + \frac{C}{(1+\mu^2)^2} + \frac{5D}{(1+9\mu^2)^2} + \frac{5G}{(9+\mu^2)^2} + \frac{H}{4(1+\mu^2)^2}\right.$$

$$+ \left(\frac{1}{2}+\rho\right)\left\{\frac{1}{4}\left(\frac{A}{\mu^4}+B\right) + \frac{2C}{(1+\mu^2)^2} + \frac{10D}{(1+9\mu^2)^2} + \frac{10G}{(9+\mu^2)^2} + \frac{H}{2(1+\mu^2)^2}\right\}\Bigg]$$

$$+ \frac{A}{4\mu^4} + \frac{C}{(1+\mu^2)^2} + \frac{9D}{(1+9\mu^2)^2} + \frac{G}{(9+\mu^2)^2} + \frac{H}{4(1+\mu^2)^2}$$

$$= \frac{A}{4\mu^4}\left[-(1+\rho)\eta\xi + 1\right] + \frac{B}{4}\left[-(1+\rho)\eta\xi\right] + \frac{C}{(1+\mu^2)^2}\left[-2(1+\rho)\eta\xi + 1\right]$$

$$+ \frac{D}{(1+9\mu^2)^2}\left[-10(1+\rho)\eta\xi+9\right] + \frac{G}{(9+\mu^2)^2}\left[-10(1+\rho)\eta\xi+1\right] + \frac{H}{4(1+\mu^2)^2}\left[-2(1+\rho)\eta\xi+1\right]$$

$$\left(\frac{pR^2}{Et^2}\frac{1}{\mu^4}\right)_0 = \frac{\xi\left\{\frac{1}{8}\eta\xi - \left(\frac{1}{2}+\rho\right)\right\}}{4\mu^4}\left[-(1+\rho)\eta\xi+1\right] + \frac{\xi\left\{\frac{1}{8}\eta\xi\right\}}{4}\left[-(1+\rho)\eta\xi\right]$$

$$+ \frac{\xi\left\{\frac{1}{2}\left(\frac{1}{2}+\rho\right)\eta\xi - \frac{1}{2}\right\}}{(1+\mu^2)^2}\left[-2(1+\rho)\eta\xi+1\right] + \frac{\frac{1}{4}\xi\left(\frac{1}{2}+\rho\right)\eta\xi}{(1+9\mu^2)^2}\left[-10(1+\rho)\eta\xi+9\right]$$

$$+ \frac{\frac{1}{4}\xi\left(\frac{1}{2}+\rho\right)\eta\xi}{(9+\mu^2)^2}\left[-10(1+\rho)\eta\xi+1\right] + \frac{\xi\left(\frac{1}{2}+\rho\right)^2\xi\eta}{4(1+\mu^2)^2}\left[-2(1+\rho)\eta\xi+1\right]$$

$= \mathcal{L}$

$$\left(\frac{pR^2}{Et^2}\frac{1}{\mu^4}\right)_0 = \xi\Bigg[-(\eta\xi)^2\Bigg\{\frac{1+\rho}{32\mu^4} + \frac{1+\rho}{32} + \frac{(1+\rho)(\frac{1}{2}+\rho)}{(1+\mu^2)^2} + \frac{5(1+\rho)(\frac{1}{2}+\rho)}{2(1+9\mu^2)^2}$$

$$+ \frac{5(1+\rho)(\frac{1}{2}+\rho)}{2(9+\mu^2)^2} + \frac{(1+\rho)(\frac{1}{2}+\rho)^2}{2(1+\mu^2)^2}\Bigg\}$$

$$+(\eta\xi)\Bigg\{\frac{1}{32\mu^4} + \frac{(1+\rho)(\frac{1}{2}+\rho)}{4\mu^4} + \frac{(\frac{1}{2}+\rho)}{2(1+\mu^2)^2} + \frac{(1+\rho)}{(1+\mu^2)^2} + \frac{9(\frac{1}{2}+\rho)}{4(1+9\mu^2)^2}$$

$$+ \frac{(\frac{1}{2}+\rho)}{4(9+\mu^2)^2} + \frac{(\frac{1}{2}+\rho)^2}{4(1+\mu^2)^2}\Bigg\} - \Bigg\{\frac{(\frac{1}{2}+\rho)}{4\mu^4} + \frac{1}{2(1+\mu^2)^2}\Bigg\}\Bigg]$$

$$= \xi\Bigg[-(\eta\xi)^2(1+\rho)\Bigg\{\frac{1}{32\mu^4} + \frac{1}{32} + \frac{(\frac{1}{2}+\rho)}{(1+\mu^2)^2} + \frac{5(\frac{1}{2}+\rho)}{2(1+9\mu^2)^2} + \frac{5(\frac{1}{2}+\rho)}{2(9+\mu^2)^2} + \frac{(\frac{1}{2}+\rho)^2}{2(1+\mu^2)^2}\Bigg\}$$

$$+ (\eta\xi)\Bigg\{\frac{1}{32\mu^4} + \frac{(1+\rho)}{(1+\mu^2)^2} + (\tfrac{1}{2}+\rho)\Bigg[\frac{1}{2(1+\mu^2)^2} + \frac{(1+\rho)}{4\mu^4} + \frac{9}{4(1+9\mu^2)^2} + \frac{1}{4(9+\mu^2)^2}$$

$$+ \frac{(\frac{1}{2}+\rho)}{4(1+\mu^2)^2}\Bigg]\Bigg\} - \Bigg[\frac{(\frac{1}{2}+\rho)}{4\mu^4} + \frac{1}{2(1+\mu^2)^2}\Bigg]\Bigg]$$

$\eta = 0.225$; $\xi = 6$; $\rho = -0.0708506$

$\eta\xi = 1.350$ $1+\rho = 0.9291494$

$(\eta\xi)^2 = 1.8225$ $\frac{1}{2}+\rho = 0.4291494$

$$\left(\frac{pR^2}{Et^2}\right)_0 = 6\Big[-1.8225 \times 0.9291494 \times 0.2142660 + 1.350\,(\overbrace{0.3279098 + 0.1111966}^{0.4391064}) - 0.2322874\Big]$$

$= \underline{-0.0139584}$

$\eta = 0.225$, $\xi = 7.5$; $\rho = -0.0852641$

$\eta\xi = 1.6875$ $1+\rho = 0.9147359$

$(\eta\xi)^2 = 2.84765625$ $\frac{1}{2}+\rho = 0.4147359$

$$\left(\frac{pR^2}{Et^2}\right)_0 = 7.5\Big[-2.84765625 \times 0.9147359 \times 0.2084215 + 1.6875\left(\begin{array}{r}0.3221444 +\\ 0.1055938\\ \hline 0.4277382\end{array}\right) - 0.2286840\Big]$$

$= \underline{-0.3733740}$

$\eta = 0.225$; $\xi = 9$; $\rho = -0.1008270$;

$(\eta\xi) = 2.025$ $1+\rho = 0.8991730$

$(\eta\xi)^2 = 4.100625$ $\frac{1}{2}+\rho = 0.3991730$

$$\left(\frac{pR^2}{Et^2}\right)_0 = 9\Big[-4.100625 \times 0.8991730 \times 0.2021693 + 2.025\,(\overbrace{0.3159192 + 0.0996901}^{0.4156093}) - 0.2247933\Big]$$

$= \underline{-1.1575557}$

h

When $\mu = 1$.

$$\left(\frac{pR^2}{Et^2}\right)_0 = \xi\Big[-(\eta\xi)^2(1+\rho)\Big\{\frac{1}{16} + \frac{3(\frac{1}{2}+\rho)}{10} + \frac{(\frac{1}{2}+\rho)^2}{8}\Big\} +$$

$$+(\eta\xi)\Big\{\frac{1}{32} + \frac{(1+\rho)}{4} + \frac{3(\frac{1}{2}+\rho)}{20} + \frac{(1+\rho)(\frac{1}{2}+\rho)}{4} + \frac{(\frac{1}{2}+\rho)^2}{16}\Big\}$$

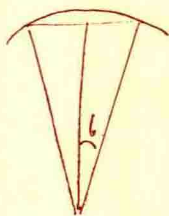
$$-\Big\{\frac{(\frac{1}{2}+\rho)}{4} + \frac{1}{8}\Big\}\Big]$$

$\eta = 0.225$; $\xi = 4$. $(1+\rho) = 0.9450611$, $(\frac{1}{2}+\rho) = 0.4450611$

$$\left(\frac{pR^2}{Et^2}\right)_0 = 4\Big[-0.810 \times 0.9450611 \times 0.2207782 + 0.900 \times (0.03125 + 0.2362653$$

$$+ 0.0667592 + 0.1051525 + 0.0123800) - 0.2362653\Big]$$

$$= 4\Big[-0.1670056 + 0.4066243 - 0.2362653\Big] = \underline{0.0054216}$$



$1 - \cos 6^\circ = 0.0055$

$$b = \frac{2\pi R}{15}$$

$$\frac{b}{t} = \frac{2\pi}{15}\,\frac{R}{t}$$

$$\frac{\sigma}{E} =$$

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